DOCUMENT RESUME

ED 103 266 95 SE 018 748

AUTHOR Kenny, Paul Francis
TITLE Effects of Group Interaction Stimulated by Competition Between Groups as a Motivating Technique in a Ninth-Grade Mathematics Classroom. Final Report.
SPONS AGENCY National Inst. of Education (DHEW), Washington, D.C. Office of Research. Basic Studies Div.
REPORT NO NIE-P-3-0321
PUB DATE 75
GRANT NE-G-00-3-0108
NOTE 115p.

EDRS PRICE MF-$0.76 HC-$5.70 PLUS POSTAGE
DESCRIPTORS Algebra; Attitudes; Doctoral Theses; *Grouping (Instructional Purposes); *Instruction; *Research; Secondary Education; *Secondary School Mathematics; Student Motivation; *Teaching Methods

ABSTRACT

The author of this report studied the differential effects on achievement and attitude toward mathematics of two types of classroom situation. Ninth-grade students were randomly assigned to traditional classrooms (lecture and class discussion of problems) or to experimental classrooms. In the latter, students were placed in groups of four members; half of all class time was devoted to teacher presentation of material, and the other half to group activities. The members of a group were encouraged to cooperate with each other by dividing assignments, peer teaching and other techniques. Groups competed against each other for test scores (groups were given scores equal to the sum of the members' scores). The investigator found no significant difference in achievement between students in traditional and experimental classrooms. Changes in attitudes toward mathematics, and the effective interaction of groups varied between groups. (SD)

U S DEPARTMENT OF HEALTH
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

BEST COPY AVAILABLE

ED103266

ABSTRACT

Title of Thesis: Effects of Group Cooperation Stimulated by Competition Between Groups as a Motivating Technique in a Ninth-Grade Mathematics Classroom

Paul Francis Kenny, Doctor of Philosophy, 1975

Thesis directed by: Dr. Ronald L. McKeen
Assistant Professor
Education

Many have voiced the opinion that a major disadvantage of many classrooms is the competitive situation in which students find themselves. They are constantly asked to excel over their peers, often at the expense of solid peer relationships. Yet, in extracurricular activities, they are asked to cooperate with one another to achieve a common goal.

Individualization of instruction attempts to alleviate competitive situations, yet does little to encourage cooperative efforts. Is it possible to organize a classroom into cooperative groups, whose members work toward the achievement of common goals? Is it possible that classrooms so organized encourage group concern for the achievement of each member of the group and that such an organization

SE 018 748

actually improves the performance of all? Is it possible that such an arrangement is particularly suited to the needs of poorly motivated, low-average achievers in mathematics, and that, for such students, attitude toward mathematics is improved?

The investigator of this study attempted to provide some answers to such questions by posing the following hypotheses:

I. Male students of low average to average mathematical ability perform significantly better through group cooperation stimulated by competition between groups than do students taught in a traditional manner.

II. A significant attitude change toward mathematics will result from group cooperation stimulated by competition between groups for male students of low average mathematical ability.

Four ninth-grade boys' mathematics classrooms were organized into groups of four members each for a period of eighteen weeks. One-half of class time was devoted to lecture by the teacher; the other half to group work. Cooperation with other members of the group was generated by: (1) group assignments--members working together on achieving conceptual and mechanical skills after explanation

by the teacher; (2) peer tutoring; (3) individual test score pooling into a group score; and (4) team spirit--a sense of accomplishment and pride for the achievement of the whole group. Competition between groups was generated through weekly assessment tasks whereby group scores were compared. A cumulative tally of scores was kept so that after eighteen weeks, the highest scoring group was awarded a prize signifying their excellence in the competition. At the same time, four additional ninth-grade boys' mathematics classrooms were conducted in a traditional manner--lecture by the teacher and class participation in problem solving.

A total of 240 ninth-grade male students from four high schools participated in the experiment. The high schools are located in Maryland, Virginia, Delaware, and Pennsylvania. All draw from populations of nearly similar economic background. The students were considered to be low average achievers in mathematics by each of the schools. Average IQ was 103. Within these restrictions, the students were randomly selected and randomly assigned to one of two levels of instruction: small group cooperation and traditional lecture.

The independent variables in the study were the two levels of instruction. The dependent variables were achievement in mathematics and attitude toward mathematics. Teacher

variability was controlled by training sessions before the experiment, a uniform syllabus, and uniform methods of presentation. In addition, four visits by the principal investigator to each of the schools verified consistency in procedures.

The Kepner Mid-Year Algebra Achievement Test was used to measure achievement after eighteen weeks. The Aiken-Dreger Mathematics Attitude Scale was administered before and after the experiment to determine treatment effects with respect to attitude. A 4 x 2 design was used to compare the four high schools and the two treatment levels. The statistic employed for the Kepner Test was the Student t ratio for a two-sample case, and for the Aiken-Dreger Scale, the Student t ratio for correlated samples. The significance level was set at .05 for both tests.

The data did not support either hypothesis. Achievement scores were not significantly higher in any of the four schools. Attitude changes occurred, but not as a result of the treatment. Questions concerning variations of the method, along with resulting hypotheses, were posed for future research.

VITA

Name: Paul Francis Kenny.

Date of birth: October 14, 1934.

Place of birth: Conception Harbor, Newfoundland.

Secondary education: Northeast Catholic High School, 1952
Philadelphia, Pennsylvania.

Collegiate institutions attended:

	Dates	Degree	Date of Degree
Niagara University	1956-59	B.S.	1959
Catholic University	1959-63	M.T.S.	1963
University of Toledo	1966-68		
Catholic University	1972-74	Ph.D.	1975

Major: Master's--Mathematics
Doctorate--Mathematics Education.

Teaching and Work Experience:
12 years mathematics instructor (high school)
2 years Assistant Principal
2 years Principal

Positions held:

Instructor, Mathematics
Father Judge High School
Philadelphia, Pennsylvania
1954-55.

Instructor, Mathematics
St. Francis High School
Toledo, Ohio
1963-68.

Positions held:

Assistant Principal and Instructor, Mathematics
Bishop Ireton High School
Alexandria, Virginia
1968-70.

Principal and Instructor, Mathematics
Bishop Ireton High School
Alexandria, Virginia
1970-72.

Instructor, Mathematics (Part-time)
Good Counsel High School
Wheaton, Maryland
1972-74.

FINAL REPORT

EFFECTS OF GROUP INTERACTION STIMULATED BY COMPETITION BETWEEN GROUPS AS A MOTIVATING TECHNIQUE IN A NINTH-GRADE MATHEMATICS CLASS

National Institute of Education Project No. 3-0321

Paul F. Kenny
Project Director

1975

The research reported herein was performed pursuant to a grant contract with the National Institute of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official National Institute of Education position or policy.

TABLE OF CONTENTS

	Page
ACKNOWLEDGEMENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	viii
Chapter	
I. INTRODUCTION	1
Theories of Cooperation and Competition	2
Competition and Cooperation in the Schools	5
Competition	5
Individualized Instruction	8
Cooperation	10
A Possible Alternative	12
Statement of Purpose	13
Summary of the Chapters	15
Footnotes--Chapter I	16
II. REVIEW OF THE LITERATURE	18
Early Studies of Competition	18
Early Studies of Cooperation	20
Cooperation and Competition	22
Later Studies of Cooperation: Supportive and Non-Supportive	23
Studies Supportive of Cooperation	23
Studies Not Supportive of Cooperation	26
An Assessment of the Claims	29

Chapter Page

Recent Experiments Involving Cooperation in Learning Mathematics 32
The Present Study 35
Summary 36
Research Hypotheses 37
Footnotes--Chapter II 39

III. EXPERIMENTAL PROCEDURE 42

Preliminary Stages 42
Research Setting 43
Sample 44
Teachers 45
Methodology 45

Group Size 47
Conducting the Group Class 48
Conducting the Traditional Class 49
Assessment Tasks 49

Instruments 50
Design and Analysis 51
Footnotes--Chapter III 52

IV. DATA ANALYSIS 53

The Weekly Assessment Tasks 53
The Statistic for the Kepner Test: *t* ratio . 59
The Statistic for the Aiken-Dreger Scale: *t* ratio 64
Summary 70

V. CONCLUSIONS, INTERPRETATIONS, AND RECOMMENDATIONS 71

Conclusions 71
Interpretation 72
Discussion 73

Variables Affecting Achievement 73
Variables Affecting Attitude 77
Cooperative Competition--An Evaluation . . 79
Alterations for Possible Follow-up Studies 79

Chapter Page

Summary 86
Footnotes--Chapter V 88

Appendix

A. SYLLABUS: ALGEBRA I (ONE SEMESTER) 89

B. AN EXERCISE IN GROUP DYNAMICS: DECISION BY CONSENSUS 92

BIBLIOGRAPHY 97

LIST OF TABLES

Table		Page
4.1	Means of Assessment Tasks for Both Classes in Each of the High Schools	54
4.2	Raw Scores of the Kepner Test for Both Classes in Each of the High Schools	61
4.3	Statistics Based on Kepner Achievement Test . .	63
4.4	Raw Scores of the Aiken-Dreger Attitude Scale for Both Classes Before and After the Experiment in Each of the High Schools	66
4.5	Statistics Based on Aiken-Dreger Attitude Scale	69

LIST OF FIGURES

Figure		Page
4.1	Means of Assessment Tasks by Condition and Chronology--Good Counsel High School	55
4.2	Means of Assessment Tasks by Condition and Chronology--Bishop Ireton High School	56
4.3	Means of Assessment Tasks by Condition and Chronology--Salesianum High School	57
4.4	Means of Assessment Tasks by Condition and Chronology--Northeast Catholic High School	58
4.5	Means of Kepner Test by Schools and Condition	62
4.6	Means of Aiken-Dreger Scale by Schools and Condition	67

CHAPTER I

INTRODUCTION

An investigation of some sociological ramifications of the educational system in existence in the United States today yields one obvious concern: the amount of competition and cooperation that is fostered by the system. It has been the experience of this investigator that competition in the schools exists to such a degree that it is often a detriment to the competitors, the students themselves. What cooperation exists is found mainly in the areas of extra-curricular activities. Many educators have expressed similar viewpoints. Some will be cited throughout this chapter.

However, before a case is constructed for the existence or non-existence of competition and cooperation in the schools, a theory of cooperation and competition should be investigated.

At the outset, this investigator accepts the premise of May and Doob[1] that competition and cooperation are primarily psychological behaviors with social ramifications. Thus, what might appear to be a competitive or cooperative

enterprise as viewed objectively by an outside observer may or may not be the same subjective interpretation of the persons involved in the enterprise. In other words, credence must be given to the proposition that in any competitive or cooperative venture, any number of motives may be present in the participants.

Thus, to establish a theory of competition and cooperation based on a psychological framework, four crucial questions must be considered. First, why do individuals compete or cooperate? Second, for what things do they compete or cooperate? Third, with what persons do they cooperate, or compete rather than cooperate, or cooperate rather than compete? Fourth, in what manner do they compete or cooperate?

Theories of Cooperation and Competition

A brief synopsis of a theory of cooperation and competition by May and Doob answers these questions in the form of four postulates presented on two levels: the social level and the psychological level.

> Postulate 5. On a social level, individuals compete with one another when: (1) they are striving to achieve the same goal that is scarce; (2) they are prevented by the rules of the situation from achieving this goal in equal amounts; (3) they perform better when the goal can be achieved in

unequal amounts; and (4) they have relatively few psychologically affiliative contacts with one another.

Postulate 6. On a social level, individuals cooperate with one another when: (1) they are striving to achieve the same or complementary goals that can be shared; (2) they are required by the rules of the situation to achieve this goal in nearly equal amounts; (3) they perform better when the goal can be achieved in equal amounts; and (4) they have relatively many psychological affiliative contacts with one another.

Postulate 7. On a psychological level, an individual competes with others when: (1) there is a discrepancy between his level of achievement and his level of aspiration; (2) his knowledge of the goal that he seeks indicates that it is limited and cannot be shared at least equally by other persons in that situation; (3) his attitudes produce within him a state in which his attitude toward competing overbalances possible conflicting attitudes toward potential competitors, toward the rules of the situation, toward cooperating rather than competing, etc.; and (4) his skill is of such a nature that under the rules of the situation he has a reasonable chance of success by competing.

Postulate 8. On a psychological level, an individual cooperates with others when: (1) there is a discrepancy between his level of achievement and his level of aspiration; (2) his knowledge of the goal that he seeks indicates that it can be reached by striving with others; (3) his attitudes produce within him a state in which his attitude toward cooperating overbalances possible conflicting attitudes toward potential cooperators, toward the rules of the situation, toward competing rather than cooperating, etc.; and (4) his skill is of such a nature that under the rules of the situation he has a reasonable chance of success by cooperating.[2]

In his initial study of cooperation and competition, Deutsch[3] expands this theory and refers to a cooperative

situation as one in which individuals perceive themselves to be "promotively interdependent" and a competitive situation as one in which individuals perceive themselves to be "contriently independent." The former specifies a condition in which goal regions for each of the individuals or sub-units in the situation are defined so that a goal-region can be entered, to some degree, by any given individual or sub-unit only if all the individuals or sub-units under consideration can also enter their respective goal-regions, to some degree.[4] The latter refers to a situation in which goal-regions for each of the individuals or sub-units in the situation are defined so that if a goal-region is entered by any individual or sub-unit, the other individuals or sub-units will, to some degree, be unable to reach their respective goals in the social situation under consideration.[5]

In a later work dealing with cooperation and trust,[6] Deutsch introduces a third situation which seems to be non-cooperative and at the same time non-competitive. This he calls "individualistic" whereby a subject was led to feel that his only interest was in doing as well as he could for himself, without regard to how well the other person did and that the other person felt the same way. He was unlikely to be focused upon benefiting or harming the other person or to perceive that the other had the intention of benefiting or

harming him. His intention toward the other and the other's intention toward him had no intrinsic source reliability in terms of whether they would be beneficial or harmful.

Competition and Cooperation in the Schools

In the light of this theoretical development concerning cooperation and competition, what can be said about the existence of cooperation and competition in the schools? If competition exists to such a degree that it is harmful to the participants, can or should it be eliminated entirely? If cooperation does not exist, or exists in relatively minimal degrees, can it be encouraged and fostered?

Competition

Does competition exist in the schools? May and Doob[7] have answered in the affirmative by stating that a curious paradox exists in the public schools of America inasmuch as the basic structure of the system is competitive, and yet the ideals of cooperation are emphasized. They point out that the competitive structure of the public schools is promoted by examinations, the emphasis on marks, the seating arrangements, preferential treatment of children, and athletic contests. Thus, a large proportion of the daily activities of the pupils is more competitive than cooperative in nature. But, at the same time, all of the human

virtues and attitudes that are favorable to cooperation are stressed.

Although these observations were made by the authors in 1937, it is this investigator's opinion that they are still valid today.

As has been noted, there are relatively few "pure" competitive situations in reality. However, the school situation is definitely competitive in the grading procedures of most educational systems in the sense that they fulfill the definitions of competition developed above. One startling observation can be deduced from Postulate 7. The fourth characteristic, "his skill is of such a nature that under the rules of the situation he has a reasonable chance of success by competing," is fulfilled by some students, but by no means all. For those students whose skill is of such a nature that they do not have reasonable chance of success, their state is considerably worse than those students who at least fulfill the minimal requirements of competition. Mouly[8] refers to this situation when he observes that overemphasis on competition almost invariably leads to indifferent performance on the part of the weaker students who cannot afford to try. For when the child encounters continuous failure, the line of least resistance, and the most intelligent, is to stop trying. Mouly also points out some

of the more obvious harmful effects of competition: a downgrading of the self-concept, an indifference toward schoolwork as a means of maintaining an image of personal adequacy in the face of poor scholastic performance, and the destruction of group loyalties and of the child's capacity to cooperate. In addition, competition can cause resentment, jealousy, and poor intragroup relations. In effect, Mouly maintains, it can negate the very values for which the school and democratic society stand.

As for those students who do have a reasonable chance of success, Kirschenbaum offers this comment:

> . . . In high school, most of us were too young or too naive to be consciously cynical about what we were doing. We had mastered the rules without realizing the destructive game we were playing. Good grades were like games won out on the ball field, and points earned in American history were almost the same as those scored in a basketball game. The idea of competition was exalted as a good thing, both in studies and in sports. My guess is that this hasn't changed much in the four years since I left Mapleton High. It's still dog-eat-dog. Yes, they put me out on the academic playing field and told me to compete, so I competed. Just like <u>every student in this auditorium</u> is competing with every other student. That's right. Look around you. Take a look at your competition. Look around you. There they are --your enemies. Don't laugh; think about it. <u>Your enemies</u>. There are several kinds of competition. One kind is when people choose to compete, according to standards they themselves have set and regard as important. I think there's a lot to be said for this kind of competition. Theoretically, everyone can be a winner. Another kind of competition is when people are <u>forced</u> to compete against one another, according to standards that are imposed upon them by others. With this kind of competition, the kind we find in

> the schools, there must be a loser for every winner. So you see, I wasn't kidding before. You are each one another's enemy. For every winner out there, there's also a loser.[9]

Competition then does exist in the schools. In an attempt to downplay the competitive atmosphere, many approaches have been tried.

Individualized Instruction

One such approach is individualized instruction. But often, individualized or personalized instruction, replete with learning packets, contracts, self-pacing, etc., can become a curious application of Deutsch's non-competitive, non-cooperative "individualistic" model, somewhat akin to the biblical "neither hot, nor cold, but lukewarm." The model is applicable to education when the student feels that his only interest is in doing as well for himself as he can, without regard to how well another student does and the other student feels the same way.

Such a concept is fraught with many shortcomings. Hyman[10] has uncovered what he calls the hidden agenda of individualization. He maintains that the reason why schools have gone to individualization is to meet the demands of critics (Silberman, Kohl, Holt, etc.) and also the demands of the students. Thus individualization strives to eliminate quite a few red herrings, among them: dull class lessons,

pace that is too slow for the superior pupils and at the same time too fast for the slow pupils, uniformity in teaching across age and ability levels, and complete dependence on the teacher for deciding what to do almost every minute of the school day. In general, Hyman concludes that much of this is accomplished, but paradoxically, the status quo is still maintained; and many fail to realize this hidden effect. For when groups are broken up, the teacher's power and authority are preserved. By isolating one pupil from another, the teacher maintains control. And in a one-to-one situation, the teacher can obviate a threat to himself.

As Hyman assesses it, some effects of the hidden agenda can be downright damaging such as the loss of group camaraderie which comes with the isolation of the pupil. He contends that the significance of this loss is beyond measure. For if the very essence of democracy is the feeling of responsibility to one's fellows which evolves from participating together in common activities, democracy cannot and does not flourish when people act in isolation. Democracy flourishes when people develop group spirit, and group spirit develops only when people share in common endeavors.

In addition, Hyman sees the individualizing of the program and the loss of group interaction as leading to the

minimizing of peer teaching. Peer teaching, whether it is intentional or unintentional, is important. Much of what each one learns is learned from peers. The person who teaches benefits as well as the person who learns. There are times when a young pupil's peers can explain and demonstrate something to him better than any trained adult can.

As Hyman expresses it:

> . . . Individualized programs of the types described here appear overtly to fulfill certain needs while covertly subverting them. To succeed in these programs, the pupil has to be able to be alone, to study alone, and to follow the directions given to him by the teacher of the packaged material. The pupil needs to be quiet, to listen, and to follow orders from an authority.
>
> In short, individualization of the types described here has certain benefits that stem from its manifest functions and certain weaknesses that stem from its latent functions. The latent functions are to maintain the power and the authority of the teacher, to continue the teacher's control of the pupils, to break up group camaraderie and peer teaching, to foster dependence and docility on the part of the pupils, and to pre-adapt pupils to the industrial bureaucracy by preparing them to work alone and follow orders.[11]

Cooperation

If students are competitors for grades, individualistic in some of the innovative programs, when are they cooperative? The answer seems to be: very infrequently. In a recent address, Jerome Bruner[12] echoed many of the sentiments that May and Doob made in 1937. He contended that there is very little organized cooperative activity

during the school years. Most joint enterprises are "extra-curricular"--social, political, or artistic. He charged that the main enterprise, studies, is similarly lacking in social cooperation. Too strong an effort on behalf of the performance of a fellow student might even be interpreted as cheating. And therein lies the anomaly in the conduct of schools: students are required to compete with each other in their studies, and yet, are urged to cooperate to the utmost in other forms of activity. In concluding his remarks, Bruner recommended that "we use the system of student-assisted learning from the start in our schools, . . . that we treat the process of mastering the culture's devices and disciplines, its tools, as a communal undertaking."[13]

There have been some spotty attempts at cooperation in the schools. One school in particular in Barbiana, Italy, is operated totally on the premise that cooperation comes first. In Letter to a Teacher, one student-teacher (between the ages of 12 and 16) makes this remark:

> Then, too, I was learning so many things while I taught. For instance, that others' problems are like mine. To come out of them together is good politics. To come out alone is stinginess. I was not vaccinated against stinginess myself. During the exams I felt like sending the little ones to hell and studying on my own. I was a boy like your boys, but up at Barbiana I couldn't admit it to myself or to others. I had to be generous even when I didn't feel like it. To you this may seem

> a small thing. But for your students you do even less. You don't ask anything of them. You just encourage them to push ahead on their own.[14]

In a postscript to this book, Robert Coles describes an experience that happened to him:

> And in Boston's ghetto, only weeks before I sat down to read this book, a ten-year-old child told me, "If you worry about the next guy, you'll never finish your own work, that's what the teacher said; but he's my brother, the next guy, only I didn't tell her."[15]

That more cooperation is needed in the schools is a concern of many. Lindgren[16] is typical of others when he argues that the skills of cooperation are far more crucial in today's world than are the skills of competition. For in the final analysis, the survival of the civilized world will depend on man's ability to learn to cooperate more effectively and to teach others how to do so. In the light of the population explosion, and the concomitant moral obligation of wealthy nations to share their resources with the poorer nations, competition can only be regarded as a more primitive and less mature approach to human relations than is cooperation.

A Possible Alternative

Is it possible to encourage cooperation within the schools? And if so, how can it be done, given the competitive atmosphere of the society in which we live? Campbell[17]

offers a solution through what he calls "the paradox of cooperative rivalry." It is achieved through group cooperation that contains both elements; that is, within-group cooperation and between-group competition. Such an arrangement can develop team play, community spirit, self-discipline, and high morale. It can intensify the urge to belong and provide a protective form of competition. However, Campbell cautions that an extreme emphasis on cooperation is not without its problems. Pupils may not learn to work alone if they depend excessively on group activities, and it may simply be the case that the learning style of some pupils is individual and independent.

In the final analysis, Campbell suggests that while the teacher's major thrust should be toward an emphasis on cooperation, healthy forms of competition can take place within cooperative types of learning activities. For ultimately, the effectiveness of the teacher will be gauged to a large extent on his ability to recognize and direct the multi-varied need levels of his students in the areas of cooperation and competition.

Statement of Purpose

The research contained within this study is the result of an experiment in which cooperative rivalry among

ninth-grade boys was conducted for a period of eighteen weeks in a mathematics classroom. The subjects were low-average achievers, the ones who most frequently have little chance of success in individual competition in school.

The experiment was undertaken so that the effects of cooperative rivalry might be measure with respect to achievement and attitude and at the same time be compared to control groups.

Thus, the purpose to which the investigator of this study addressed himself was to examine a teaching technique which would be an alternative to individual competition and individualized instruction. This would be accomplished in the following manner:

1. By organizing a classroom into small groups which would provide within-group cooperation induced by between-group competition.
2. By describing the procedure whereby classes so organized were conducted.
3. By measuring the effects of cooperative rivalry with respect to achievement and attitude.
4. By testing the hypotheses that significant achievement scores and positive attitude direction would be the results of such an arrangement.

Summary of the Chapters

Chapter I supplies a background for a statement of the problem. Chapter II contains a review of the literature relevant to cooperation and competition in the classroom. The procedure used in the experiment is described in Chapter III. Chapter IV presents the data obtained for achievement and attitude. And finally, interpretation of data, conclusions, and recommendations are reported in Chapter V.

Footnotes--Chapter I

[1]Mark A. May and Leonard W. Doob, Competition and Cooperation, a Report of the Sub-Committee on Competitive-Cooperative Habits (Social Science Research Council, 1937).

[2]Ibid., pp. 5-6.

[3]Morton Deutsch, "A Theory of Cooperation and Competition," Human Relations, 2 (1949), 129-51.

[4]Ibid., pp. 131-32.

[5]Ibid., p. 132.

[6]Morton Deutsch, "Cooperation and Trust: Some Theoretical Notes," in Nebraska Symposium on Motivation, 1962, ed. by Marshall R. Jones (Lincoln, Neb.: University of Nebraska Press, 1962), pp. 306-307.

[7]May and Doob, Competition and Cooperation, p. 82.

[8]George J. Mouly, Psychology for Effective Teaching (2d ed.; Atlanta, Ga.: Holt, Rinehart and Winston, Inc., 1968), p. 355.

[9]Howard Kirschenbaum, Rodney W. Napier, and Sidney B. Simon, Wad-ja-Get? The Grading Game in American Education (New York: Hart Publishing Co., Inc., 1971), pp. 81-83.

[10]R. T. Hyman, "Individualization: The Hidden Agenda," Elementary School Journal, 73 (May 1973), pp. 412-23.

[11]Ibid., pp. 415-16.

[12]Jerome Bruner, "The Continuity of Learning," Saturday Review of Education, 1 (February 10, 1973), 24.

[13]Ibid.

[14]The School Boys of Barbiana, Letter to a Teacher, trans by Nora Rossi and Tom Cole (New York: Random House, Inc., 1971), p. 8.

[15] Ibid., p. 159.

[16] Henry Clay Lindgren, Educational Psychology in the Classroom (3d ed.; New York: John Wiley & Sons, Inc., 1967), p. 160.

[17] Lloyd Campbell, "Cooperation or Competition," Clearing House, 47 (May 1973), 563-64.

CHAPTER II

REVIEW OF THE LITERATURE

An examination of the literature pertinent to cooperation and competition leaves the impression that the results are ambiguous at best; the competition theorists claiming that competition induces productivity, while the cooperation theorists make similar claims for cooperative efforts.

This chapter will provide an overview of the various experiments dealing with cooperation and competition in the classroom, and an attempt will be made to analyze why such conflicting claims can co-exist.

Early Studies of Competition

Early studies of competition in the classroom seem to confirm one major result: productivity is increased, but the quality of the work produced is not. Thus, Chapman and Feder's[1] investigation of competition between individuals in the classroom in 1917 yielded the observation that the incentive (competition) exerted a considerable effect on the amount of the product. In 1925, Whittemore[2] performed an

experiment with individuals in a competitive condition and in a non-competitive condition, and with groups competing with other groups. He reported that while all subjects turned out more work when competing than when not, they also did poorer work. On a semi-mechanical task, competition tended to emphasize speed rather than quality.

Yet, in 1927, Hurlock[3], seems to contradict these findings in her report on an experiment wherein fourth and sixth graders were divided into two groups: control and rivalry. The rivalry condition resulted from a division of the group into two competing subgroups. Arithmetic tests were the competition devices and took place on five sucessive days. She found that the mean scores of the rivalry group exceeded those of the control group on every day of the experiment except the first.

Then, in 1928, Sims[4] made the observation that individual motivation (competition) is vastly superior to group motivation (competition); and group motivation is only slightly superior to no motivation other than that which comes incidentally in learning. He based these conclusions on two experiments. In one, his sample consisted of 126 college sophomores who were assigned to one of three sections: control, group-motivated, and individually-motivated. The task developed was one involving substitution of digits

for letters and graded on the basis of the number of substitutions made per minute. The experiment was conducted three times a week for a total of twelve practice periods. The control section had no motivation other than that which came incidentally. The group-motivated section was divided into two subgroups, one competing against the other. And in the individually-motivated section, the students were organized into pairs, with the two members competing against each other. Another experiment, having a different task but utilizing the same conditions, produced the same results: individual motivation was vastly superior to group motivation; and group motivation was only slightly superior to no motivation.

An analysis of these early experiments shows that the tasks used were of slight duration and rather mechanical in nature. And except for Sims, all investigators based their conclusions on measuring instruments that were utilized only while the experiments were in progress. Sims did use a posttest to measure effects.

Early Studies of Cooperation

In 1926, a new trend began to develop with investigators concentrating on measuring the effects of cooperation. Barton[5] set up an experiment whereby he could measure group

activity versus individual effort in solving problems in first-year Algebra. In one section, he assigned his students new problems to be solved as individuals; and in another, new problems to be solved by class discussion. Posttests favored class discussion for problem-solving in Algebra.

Marjorie Shaw[6] made a similar contribution in 1932. She set out to compare the ability of individuals and cooperating groups of four persons in solving complex problems. The groups were roughly equated so that no one group was composed of four superior individuals. Results indicated that groups seemed assured of a much larger proportion of correct solutions than individuals. She concluded that rejection of incorrect suggestions and checking of errors in the group accounted for the results.

In this same vein, Klugman[7] showed in 1944 that two heads were better than one in the solution of twenty arithmetic problems graduated in difficulty. In his experiment, two children working together did significantly more problems correctly but in a longer period of time than each child working alone.

Cooperation and Competition

A major contribution to the literature concerning experiments in cooperation and competition was made by Deutsch[8] in 1949. A total of thirty-four hypotheses were tested, the main one being that "individuals who are exposed to the cooperative social situation would perceive themselves to be more promotively interdependent with respect to goal, locomotions, facilitations, and similar matters, than will individuals who are exposed to the competitive social situation."[9] Other hypotheses posited the higher ranking of cooperative groups over competitive groups with respect to organization, motivation, communication, orientation, group productivity and interpersonal relations.

Deutsch initiated his experiment by selecting fifty volunteers who were enrolled in a course in Introductory Psychology at the Massachusetts Institute of Technology. The volunteers were formed into ten groups; the groups were paired and by a random procedure, one of each pair was assigned to the "cooperative" treatment and the other to the "competitive" treatment. The five "cooperative" groups were instructed that each week they would be given a puzzle to solve as a group. The puzzles would be tests of the group's ability to do clear, logical thinking. Each week the groups would be ranked by observers and at the end of five weeks,

the group with the best average rank would be excused from one term paper and receive an automatic H for that paper. (An H at M.I.T. is the highest grade obtainable.)

The five "competitive" groups were also instructed that they would be given weekly puzzles to solve, but they would be ranked as individuals. The highest ranking individual in each group would receive the same reward as that described for the highest ranking "cooperative" group.

Four observers used measuring instruments drawn up by Deutsch, a function observations sheet and over-all rating scales. The experimental findings gave support to most of the hypotheses. With respect to "amount of learning," no significant differences were found, although there was a trend in favor of "cooperative" groups.

Later Studies of Cooperation: Supportive and Non-Supportive

The literature of the 50's, 60's, and 70's can be classified into two types: experiments which are supportive of cooperative efforts and those which are not.

Studies Supportive of Cooperation

In 1952, Grossack[10] found that cooperation may be considered a determinant of group cohesiveness since cooperative groups showed significantly more cohesive

behavior and acceptance of pressures toward uniformity than did competitive groups. He also concluded that the frame of reference of an individual will determine his expectations of others; that is, individuals who perceive themselves as cooperative will tend to expect cooperative behavior from one another; while individuals who perceive themselves as competitive will tend to expect competitive behavior from others.

In 1958, Marvin Shaw[11] isolated some motivational factors in cooperation and competition by using a task which was intrinsically interesting to the subject and by making the performance dependent upon the subject's own efforts regardless of whether he believed himself to be in a cooperative or in a competitive situation. He did this by performing two longitudinal experiments involving a tracking task. In the first, three experimental conditions were created: (1) the subject perceived himself to be in a cooperative situation and actually was through the collaboration of a confederate of the investigator; (2) the subject perceived himself to be in a competitive situation, and actually was; and (3) the subject worked alone. The results showed that the cooperative situation was the most effective condition with respect to time and integrated error. The competitive situation was the least effective and the

individual situation fell between the two. In the second experiment, the same experimental conditions were set up as far as the subject's perception was concerned. However, in the cooperative and competitive situations, the confederate actually contributed nothing. The results for this experiment were similar to the first with the exception that differences in performance between the individual and the cooperative situation were not found.

The low performance in the competitive situation in both experiments according to Shaw was due to the fact that competitive situations arouse stronger motivation to achieve than do cooperative situations, but this stronger motivation results in poorer performance; the reason being that performance was measured by accuracy scores which were particularly susceptible to disruptive responses introduced by the energizing component of motivation and to interference by task-irrelevant responses resulting from threat to self-esteem.

In 1966, Julian, Bishop, and Fiedler[12] demonstrated that cooperation which results from intergroup competition promotes close interpersonal relations among group members and improves morale and adjustment.

Another result of intergroup competition was reported by Barrish, Saunders, and Wolf[13] in 1969. Out-of-seat and

talking-out behaviors were studied in a regular fourth-grade class that included several "problem children." The class was divided into two teams to play a game. Out-of-seat and talking-out behavior by an individual resulted in a mark on the blackboard which meant loss of privileges by all members of the student's team. Privileges included extra-recess, first to line up for lunch, time for special projects, as well as winning the game. Reliable effects were reported since out-of-seat and talking-out behaviors changed maximally only when the game was applied.

Studies Not Supportive of Cooperation

In 1960, Hudgins[14] made the distinction of measuring group performance while the experiment was being conducted and measuring again with a posttest after the experiment had been concluded. Working with 128 fifth-grade students selected in equal numbers from each of four public schools in the city of St. Louis, Hudgins set out to test two hypotheses: (1) problem-solving experience in a group improves individual ability more than does individual experience; and (2) individual ability to solve arithmetic problems improves as a result of specifying the steps involved in arriving at a solution. To do this, he used two phases of an experiment. In the first phase, some students worked in four-member

groups and others worked individually. The results of this phase indicated that students who worked in groups made higher scores than those who worked individually. In the second phase of the experiment, all students worked individually so that significant differences could be measured. The second phase resulted in no significant improvement in the problem-solving performance of the former students because of the group experience; nor was the second hypothesis supported--there was no improvement in ability to solve arithmetic problems as a result of specifying the steps involved in arriving at a solution.

Group and individual problem solving in terms of creativity was investigated by Banghart and Spraker[15] in 1963. Group make-up was determined by "interchanging information and helping one another."[16] Individuals could not exchange information or help one another. In the procedure, the teacher discussed the topic under consideration, did not explain a method of solution, but encouraged the students to find a method for themselves. "Creativity" was promoted by: (1) encouraging unique methods of solving problems; (2) asking for several methods of solving a single problem; and (3) encouraging independent generalization of these various methods of solution. A "creativity" test was composed of mathematical problems with instructions to solve

each in as many ways as possible. A t-test was used to determine a significant difference in the creativity scores. The t-test was not significant. The authors concluded: "The contribution of the group has been overly emphasized. . . . On the contrary, there seems to be a consistent, if slight, advantage to solving problems alone."[17]

Julian and Perry[18] suggested a need for qualifications of the usual generalizations drawn from previous studies of cooperation and competition as a result of an experiment conducted in 1967. The authors wished to distinguish between three conditions: (1) individual competition, (2) group competition, and (3) pure group cooperation. In the individual competition, grades were assigned on an individual basis, with the best papers receiving A, the next best, B, etc. In the group competition, grades were assigned on a curve, with members of the best group receiving A, etc. In the pure cooperative group, grades were assigned on the basis of number of points the team earned, where each member of those teams which obtained 90 per cent of possible points received an A, etc. The experiment lasted for one week in a psychology lab. The results revealed that (1) competition leads to greater productivity; (2) both the individual and the group competition conditions were significantly more productive than the pure cooperative condition; and (3) both

the quality and quantity of group performance on the lab exercise favored the individual competition with the pure cooperative groups producing the poorest results.

An Assessment of the Claims

The experiments reported above suggest a state of ambiguity in the results of measuring competition and cooperation. Several explanations can be offered. One tack is to examine the objectives of the investigators in undertaking the experiments. When the main concern is process-oriented results, cooperative efforts seem to do very well --more promotive interdependence; greater organization; more communication; motivation of a more productive type; more cohesive behavior; closer interpersonal relations; greater peer tutoring; greater perceived mutual concern. When the main concern is product-oriented results, cooperative efforts do not fare so well--no improvement in problem-solving ability; no greater creativity as a result of group experience, no greater performance levels in quantity and quality. Thus the findings should be viewed through the prism of product or of process.

Another possible explanation of the disparity of results has been offered by Miller and Hamblin.[19] They suggested that the strength and the direction of the effects

of cooperation/competition are strongly influenced by the extent to which the group has an interdependence task. In an experiment, they confirmed their hypotheses that under conditions of high task interdependence, a strong negative relation exists between the productivity of a social system and the degree of differential rewarding for relative achievement; and under conditions of low task interdependence, the positive relation between productivity and the degree of differential rewarding is at best very weak.

Thus, an obvious question concerning the experiments reported should be asked. How much interdependence or interaction existed within the groups being tested?

Thompson[20] has tackled this question in his assessment of the reported effects of cooperation and competition. He made a distinction between an "individual condition" and the "cooperative condition"; the latter being subdistinguished into the "joint-cooperative" condition and the "associate-cooperative" condition. An individual condition was described as one in which pupils do not have the opportunity to engage in interaction relevant to the accomplishment of the learning task. The cooperative condition was described as one which calls for some form of interaction between pupils. In the "joint-cooperative" condition, the group, not each individual, is required to produce one result

for the learning task. In the "associate-cooperative" condition, pupils have the opportunity of interacting, but each pupil is required to produce his own result.

With respect to the experiments described in this chapter, no significant results have been reported for product-oriented investigations, regardless of whether the condition was joint-cooperative or associate-cooperative.

However, Thompson also made the point that in either condition, an analysis must be made of the amount of interaction taking place. Thus he distinguished two types of learning tasks. In one, the task can be performed either with or without cooperative interaction taking place. But another type of learning task can be accomplished only by means of cooperative interaction between pupils. As examples of this type of task, he cited drama and many simulation games. He listed two studies that utilized the latter type of task, one by Baker[21] and one by MacKinnon.[22]

Baker's study compared a lecture and discussion history class with one which combined lecture and discussion with a simulation game where pupils played roles of leading officials in various parts of the United States during a historical period. An immediate posttest showed the statistically significant superiority of the simulation class over the lecture-discussion class. However, a retention test

administered six weeks later showed a smaller and unreliable difference between the two classes.

MacKinnon's study of first-year reading instruction showed that a condition where the teacher supervised a group of pupils in which each pupil took turns at reading while the others followed produced greater learning than a condition wherein individual children practiced reading alone.

Thompson concluded his assessment with the observation that where it is known that cooperative interaction has in fact taken place, there is evidence of facilitation of pupil learning.

Recent Experiments Involving Cooperation in Learning Mathematics

Several experiments in which the main thrust has been the effects of cooperative interaction among students in mathematics classes have been conducted within the last two years.

Turner[23] applied a limited use of small groups as a means of handling large classes in Freshman calculus. He taught a large class of students by the lecture-discussion method three days a week, and for the remaining two days, the class was divided into small groups of 12 to 20 students each, with a senior math major acting as a teaching assistant and in charge of each group. The small groups were further

subdivided into groups with three students working together.

At the same time, a control class had lecture-discussion for five days a week. No significant differences in student achievement on examinations were found.

McKeen[24] and Eisenberg[25] sought to generate a behaviorally stated learning hierarchy in a selected topic in calculus through the use of cooperative interaction within three groups of four students each. The hierarchy was generated through successive identification of tasks that had to be mastered before a solution to the original problem could be achieved.

The hierarchy was tested on a second population of students, half of which were classified as high in academic achievement, and half as low in academic achievement.

There was a high incidence of valid dependency hypotheses with respect to the data collected for the high achievers and a low incidence for the low achievers.

Davidson's[26] pilot study of the small group-discovery method of instruction in calculus raised many important questions and offered resulting hypotheses concerning the nature of small group learning through discovery. He advocated and applied the use of small groups as a replacement, rather than a supplement, for the lecture in calculus. In addition, the discovery method was the prime instrument of

learning. No textbooks in the usual sense were used; instead, notes were prepared by the teacher, and questions for investigation were formulated by him during five to ten minute discussions in class. These served as catalysts for group discovery in proving theorems and developing techniques for solving various classes of problems. Cooperation within groups was stressed and between-group competition was discouraged.

Although no significant achievement scores for the students in the small group class were reported on a final examination in comparison with a control class, they did score slightly better, and an open-ended questionnaire showed that the small group class had either positive or non-negative effects upon student interest in mathematics and estimate of problem-solving skill.

One of the questions which Davidson raised as a result of his study was: "Can a small group approach be used in mathematics instruction at all levels, from elementary school through graduate school? What would be the effects upon the learning of mathematics, the qualify of interpersonal relationships, and a host of other pertinent variables?"[27]

In 1973, DeVries and Edwards[28] made the contribution that using games in a seventh-grade mathematics class

created greater student peer tutoring, less perceived difficulty, and greater satisfaction with the class. Using student teams positively altered classroom process by creating greater student peer tutoring and greater perceived mutual concern and competitiveness in the classroom. And a games teams combination resulted in greater peer tutoring than either games or teams alone.

The Present Study

The investigator of the study reported here used a combination of lecture and small group learning in Algebra for a population of ninth-grade boys. The subjects did not have the mathematical maturity of college students, nor were they considered to be high achievers in mathematics. Consequently, adaptations of small group learning seemed appropriate. Thus, cooperation within groups was generated primarily by competition between groups, an approach which would be more in keeping with the natural inclinations of boys at that age level. In other words, a class of ninth-grade boys who were only slightly motivated, or not motivated at all, to learn mathematics might adapt to cooperative efforts with greater ease, if they were spurred on by competition between groups. The work of DeVries and Edwards, reported above, lends credence to this conjecture. In addition, only half

the class time was devoted to group work. The first half of class time consisted of a lecture by the teacher, including explanation and illustration, since the chances for low average to average ability students, with low motivation, to learn solely by discovery seemed questionable.

Summary

Early studies in competition both for individuals and for groups yielded inconclusive results. Studies involving cooperation, however, showed that cooperative efforts are superior to individual efforts. One study of cooperation and competition confirmed the hypothesis that the cooperative situation produced greater promotive interdependence. Other studies demonstrated that process-oriented hypotheses were supported for cooperative conditions, whereas product-oriented hypotheses were not.

It was demonstrated that task interdependence has a bearing on the results as does the amount of interaction taking place between pupils in joint-cooperative or associate-cooperative conditions.

In recent experiments with group learning in mathematics, promising, but not significant, results were reported. The main thrust of the experiment conducted by this investigator was the effects of group competition which

combined lecture and group work, a technique which seemed appropriate to age, achievement, and motivational factors of the subjects involved.

With respect to the distinctions made by Thompson, the group condition in this experiment was associate-cooperative (pupils had the opportunity of interacting, but each pupil was required to produce his own result); and the tasks in mathematics were intrinsically such that they could be performed with or without cooperative interaction taking place. However, even though the tasks were not of such a nature as to demand cooperative interaction, an environment for interaction was created by the investigator. In addition, both process- and product-oriented results were hypothesized.

Research Hypotheses

The product-oriented hypothesis was that male students of low average to average mathematical ability perform significantly better as a result of group competition than do similar students taught in a traditional manner. The process-oriented hypothesis was that attitude toward mathematics would be significantly changed in a positive direction for male students of low average to average mathematical ability who have experienced group competition as a major

component of mathematics instruction, and that no attitude change would occur in a traditionally-taught class.

Footnotes--Chapter II

[1]J. Crosby Chapman and R. B. Feder, "The Effect of External Incentives on Improvement," Journal of Educational Psychology, 8 (1917), 469-74.

[2]Irving C. Whittemore, "The Influence of Competition on Performance: An Experimental Study," Journal of Abnormal and Social Psychology, 19 (1925), 236-54.

[3]Elizabeth B. Hurlock, "The Use of Group Rivalry as an Incentive," Journal of Abnormal and Social Psychology, 22 (1927), 278-90.

[4]Verner Martin Sims, "The Relative Influence of Two Types of Motivation on Improvement," Journal of Educational Psychology, 19 (1928), 48-84.

[5]W. A. Barton, "Group Activity Versus Individual Effort in Developing Ability to Solve Problems in First-year Algebra," Educational Administration and Supervision, 12 (1926), 512-18.

[6]Marjorie E. Shaw, "A Comparison of Individuals and Small Groups in the Rational Solution of Complex Problems," American Journal of Psychology, 44 (1932), 491-504.

[7]Samuel F. Klugman, "Cooperative Versus Individual Efficiency in Problem-Solving," Journal of Educational Psychology, 35 (1944), 91-100.

[8]Morton Deutsch, "An Experimental Study of the Effects of Cooperation and Competition upon Group Process," Human Relations, 2 (1949), 199-232.

[9]Dorwin Cartwright and Alvin Zander, eds., Group Dynamics: Research and Theory (Evanston, Ill.: Row Peterson and Company, 1958), p. 322.

[10]M. M. Grossack, "Some Effects of Cooperation and Competition upon Small Group Behavior," Journal of Abnormal and Social Psychology, 49 (1954), 341-48.

[11]Marvin E. Shaw, "Some Motivational Factors in Cooperation and Competition," Journal of Personality, 26 (1958), 155-69.

[12]James W. Julian, Doyle W. Bishop, and Fred E. Fiedler, "Quasi-Therapeutic Effects of Intergroup Competition," Journal of Personality and Social Psychology, 3 (1966), 321-27.

[13]Harriet H. Barrish, Muriel Saunders, and Montrose M. Wolf, "Good Behavior Game: Effects of Individual Contingencies for Group Consequences on Disruptive Behavior in a Classroom," Journal of Applied Behavior Analysis, 2 (1969), 119-24.

[14]B. B. Hudgins, "Effects of Group Experience on Individual Problem Solving," Journal of Educational Psychology, 51 (1960), 37-42.

[15]Frank W. Banghart and Harold S. Spraker, "Group Influence on Creativity in Mathematics," Journal of Experimental Education, 31 (1963), 257-63.

[16]Ibid., p. 259.

[17]Ibid., p. 262.

[18]James W. Julian and Franklyn A. Perry, "Cooperation Contrasted with Intra-Group and Inter-Group Competition," Sociometry, 30 (1967), 79-90.

[19]L. Keith Miller and Robert L. Hamblin, "Interdependence, Differential Rewarding, and Productivity," American Sociological Review, 28 (1963), 768-78.

[20]G. Brian Thompson, "Effects of Cooperation and Competition on Pupil Learning," Educational Research, 15 (1972), 28-36.

[21]E. H. Baker, "A pre-Civil War Simulation for Teaching American History," in Simulation Games in Learning, ed. by S. S. Boocock and E. O. Schild (Beverly Hills, Calif.: Sage, 1968).

[22]A. R. MacKinnon, How Do Children Learn to Read? (Toronto, Can.: Copp Clark, 1959).

[23]V. D. Turner, C. D. Alders, F. Hatfield, H. Croy, and C. Sigrist, "A Study of Ways of Handling Large Classes in Freshman Mathematics," American Mathematical Monthly, 73 (1966), 768-70.

[24]Ronald L. McKeen, "A Model for Curriculum Construction through Observations of Students Solving Problems in Small Instructional Groups" (unpublished doctoral dissertation, University of Maryland, College Park, 1970).

[25]T. A. Eisenberg, "The Integration of Modified Learner-Generated Sequences into the Development of a Behaviorally Stated Learning Hierarchy, as Applied in Mathematics Curricular Construction" (unpublished doctoral dissertation, University of Maryland, College Park, 1970).

[26]Neil A. Davidson, "The Small Group-Discovery Method of Mathematics Instruction as Applied in Calculus" (unpublished doctoral dissertation, University of Wisconsin, 1970).

[27]Ibid., p. 332.

[28]David L. DeVries and Keith J. Edwards, "Learning Games and Student Teams: Their Effects on Classroom Process," American Educational Research Journal, 10 (1973), 307-18.

CHAPTER III

EXPERIMENTAL PROCEDURE

In this chapter, the various components of the experiment are described as well as the procedure developed in carrying it out. Thus, this chapter contains a report on preliminary stages, research setting, sample, teachers, methodology, instruments, and design and analysis.

Preliminary Stages

During the summer of 1973, planning was begun so that the experiment would be ready for implementation in September of the same year. Since it was determined that a full semester (18 weeks) would be given to the experiment, necessary permissions were obtained from the principals of the high schools involved. These permissions were readily granted.

The investigator then drew up a syllabus of the semester's work. Since four high schools in four different geographic areas were the locales of the experiment, it was decided that a uniform syllabus would eliminate uncontrolled

variability with respect to subject matter. The syllabus is reproduced in Appendix A.

An exercise in group dynamics suitable for cooperative efforts was decided upon. This exercise is reproduced in Appendix B.

Finally, the investigator met with the three cooperating teachers and explained the purpose of the experiment and the need for uniform procedures.

All of this was completed by September of 1973, and the experiment began.

Research Setting

The setting for the experiment was provided by four high schools, all of which are Catholic private or diocesan schools for boys. The schools are: Good Counsel High School, Wheaton, Maryland; Bishop Ireton High School, Alexandria, Virginia; Salesianum High School, Wilmington, Delaware; and Northeast Catholic High School, Philadelphia, Pennsylvania. All four high schools draw from populations of nearly similar economic background--the first three being more equal in populations: suburban, upper-middle class; the fourth, Northeast Catholic: middle class city dwellers.

These schools were selected because of ease in obtaining permissions to conduct the experiment. Three of

the schools are conducted by a religious order of priests, of which the investigator is a member. The fourth was known to the investigator because of part-time teaching there.

Sample

The sample consisted of 240 ninth-graders from the four high schools. The students were considered to be low-average achievers in mathematics. Entrance tests used by the schools and grades from the elementary schools were the determining factors of this classification. It was presumed that IQs would range from 90 to 100. However, this was the case in only one of the schools: Northeast Catholic, which had an average IQ of 93 for the students selected. The students selected at Good Counsel averaged 103 in IQ. The average IQ of the students selected at Salesianum was 103; and at Bishop Ireton, the average IQ was 113.

Since there was no significant difference in IQs for the experimental group and the control group in each high school, the internal validity of the experiment was not confounded.

In each high school, the students were randomly assigned by computer to one or two conditions: the experimental class, hereinafter designated as "group," and the control class, designated as "traditional."

The Kepner Mid-Year Algebra Achievement Test, one of the measuring instruments, was given to both classes in each high school, not as a pretest, but as a way of verifying that random selection had indeed been carried out. In each high school, there was no significant difference in the scores of the two classes and thus randomness was assured.

Teachers

Four male teachers, the investigator and three others, conducted the experiment, one in each of the high schools. All were qualified by the states in which they taught to teach mathematics and had a minimum of three years teaching experience prior to the experiment.

Methodology

During the first week of the experiment, the following chronology occurred. On the first day of school, all students were given the Kepner Test to insure randomness as has been noted: for the group class, the Kepner Test also served as a means of assigning students to groups so that the higher achieving students would not be in the same group. Since the hypotheses envisioned cooperation and peer tutoring, heterogeneous ability by group seemed a more desirable procedure.

The Aiken-Dreger Mathematics Attitude Scale was administered to all students on the second day as a pretest.

By the third day, group make-up had been determined by the teacher and the students in the group class were informed that they were assigned to a group of four students who would work as a unit for the remainder of the semester. The basis for working as a unit would be group assignments, either done in class, where each student was expected to participate, or done outside of class, again with each student participating. Peer tutoring was encouraged. If one student in the group was falling behind, it was the group's responsibility to come to his assistance. Team spirit was also emphasized, since the students were told that not only would they work in groups, but each group would be competing against other groups. A running tally would be kept of group scores on the basis of frequent tests, and at the end of the semester, the highest scoring group would receive an award. The award decided upon was a Panasonic clock-radio for each member of the group.

The remainder of the class on this day was devoted to a review of eighth-grade arithmetic. In groups, the students were instructed to test one another in simple arithmetic problems.

An exercise in group dynamics (Appendix B) was given to the group class the next day. The purpose of the exercise was to give the students an opportunity to meet each other in a group situation, and to experience a condition where the pooling of individual efforts was necessary. The exercise entails decision making, and usually a group leader emerges, but this was incidental to the main purpose: getting students to work together. From an observational point of view of all four teachers, it was successful.

Group Size

The group size was determined to be four, since researchers have indicated that interaction and peer tutoring tend to operate optimally in a group of five or less. Thelen's principle of least group size, for example, advanced the theory that the nature of the task and the smallest number of students who have among them the necessary skills should dictate the size of the group.[1] Hare[2] favored five members for discussion groups, along with Bales[3] and Slater.[4]

Davidson[5] decided upon four-member groups for his study of small group-discovery in calculus. Because of the special nature of a mathematical work group, he rejected the five member group for the following reasons: it would allow

a minimal amount of individual participation, could conceivably lead to within-group competition, and might force a "crowding out" of one member of the group. He felt that there was no clear case for choosing between a three member group and a four member group. Class size and the need for adequate attention to each group finally resulted in the decision for four member groups.

The investigator of this study chose four member groups for much the same reasons as Davidson. The groups would not be required to make decisions, but through interacting with one another, develop conceptual and mechanical skills in mathematics. In addition, class size (thirty students) clearly mitigated against three member groups.

Conducting the Group Class

The teacher began the class by reviewing work from the previous day or by going over the homework assignment. During this time, the members of each group worked together and checked one another's work. (The class was arranged so that the members in a group could sit in a circular arrangement whenever group work was desired.) The teacher would then begin a twenty-minute lecture on new material. For the rest of the class, which lasted for approximately twenty-five more minutes, the groups began an assignment given by the

teacher. The teacher acted as an observer during this period. If a particular member of a group had a question, he was directed to seek the answer from other group members. When no one in the group could arrive at a correct answer, the teacher sat down with the group and demonstrated a solution. It became obvious after several weeks of the experiment that certain individuals were more responsive to group work than others, since the amount of interaction varied from group to group. The interaction was judged by the give-and-take within the group. It was also judged by the extra-class activity of a particular group, such as working together outside of class in a study center.

Conducting the Traditional Class

This class was conducted by the teacher with a lecture for twenty minutes, and individual work on assignments for approximately twenty-five minutes. During this time, the teacher walked about the classroom and offered individual help where needed.

Assessment Tasks

Assessment tasks were given to both classes, traditional and group, on an average of every eight days. In the traditional classes, they served as assessment devices only. In the group classes, they served as assessment tasks as

well as competition devices. During any particular test, each student in the group class had to work on his own. Group means were then computed and on the basis of a point system ranging from 8 to 1, groups were scored in comparison to other groups. Thus, the group with the highest average received 8 points; the lowest, 1 point. A cumulative tally of points was kept for the entire semester.

It might be noted that interest in the competition was quite intense. When test papers were returned, group members compared solutions. It was not uncommon to see peer pressure produce more desirable results from a lagging group member in the next test.

No one group emerged as a clear winner of the competition until the last few weeks of the semester. Results of the assessment tasks as well as comparisons between the group class and the traditional class in each high school are presented in Chapter IV.

Instruments

The Kepner Mid-Year Algebra Achievement Test was used as the measuring instrument for the hypothesis that significant achievement would result. This test is published by the Bureau of Educational Research and Service at the University of Iowa. Two reliability coefficients

using the Kuder-Richardson procedure were given for the test. Formula KR-20 yielded a coefficient of .89, and Formula KR-21 gave a coefficient of .87.

This test was administered to all students in the sample during the period of January 14 to January 25, 1974. This was a time period which immediately followed the 18-week semester. Competition between groups had ceased by this time and group winners had been declared.

The Aiken-Dreger Mathematics Attitude Scale was given as a pretest during the first week of the semester and again as a posttest during the same time period when the Kepner test was given.

Design and Analysis

A 4 x 2 design consisting of the four schools and the two treatment levels was used to describe experimental effects. It was decided to use this design since teacher variability could not be completely controlled. Thus, for each school, experimental effects were treated separately.

In measuring the results of the Kepner Test, the statistic used was the Student t ratio for a two-sample case. The Student t ratio for correlated samples was employed to measure the results of the Aiken-Dreger Mathematics Attitude Scale. The significant level for both was set at .05.

Footnotes--Chapter III

[1]H. A. Thelen, "Group Dynamics in Instruction: Principle of Least Group Size," School Review, 57 (1949), 139-48.

[2]A. Paul Hare, "A Study of Interaction and Consensus in Different Sized Groups," American Sociology Review, 17 (1952), 261-67.

[3]R. F. Bales, "Small Group Theory and Research," in Sociology Today: Problems and Prospects, ed. by R. K. Merton, L. Broom, and L. S. Cottrell, Jr. (New York: Basic Books, 1958), pp. 293-305.

[4]P. E. Slater, "Contrasting Correlates of Group Size," Sociometry, 21 (1958), 129-39.

[5]Neil A. Davidson, "The Small Group-Discovery Method of Mathematics Instruction as Applied in Calculus" (unpublished doctoral dissertation, University of Wisconsin, 1970), pp. 156-59.

CHAPTER IV

DATA ANALYSIS

This chapter contains data obtained from the weekly assessment tasks, the Kepner Mid-Year Algebra Achievement Test, and the Aiken-Dreger Mathematics Attitude Scale. Statistical analysis is reported for the latter two instruments, not for the assessment tasks. Figures illustrating the results are also included. Interpretations of the findings have been deferred until Chapter V.

The Weekly Assessment Tasks

As reported in Chapter III, the assessment tasks were administered to both classes on an average of one every eight days. The means for each school for group and traditional classes are presented in Table 4.1. Figures 4.1-4.4 illustrate these results.

No statistical analysis was made of assessment task results; however, Figures 4.1 and 4.2 give an indication of what might be expected when the post-experimental measuring instrument was applied--no significant differences in achievement. Figure 4.3, on the other hand, suggests that

significant differences might be found in a posttest in favor of the group class, while Figure 4.4 suggests that significant differences might be found in the opposite direction.

Conclusions concerning treatment effects will not be discussed at this point, but in Chapter V, after all data have been presented.

Table 4.1

Means of Assessment Tasks for Both Classes in Each of the High Schools

Good Counsel		Bishop Ireton		Salesianum		Northeast Catholic	
Trad	Group	Trad	Group	Trad	Group	Trad	Group
89	84	81	79	70	76	65	67
82	81	82	82	73	78	60	54
83	78	80	80	56	64	54	43
81	80	82	83	66	73	65	45
70	74	72	75	67	76	51	46
65	75	78	78	71	78	56	5?
78	75	85	86	60	67	56	52
74	79	79	80	50	60	63	58
79	75	89	89	60	64	57	53
70	68	70	76	70	80		
72	76	[illegible]	93	70	70		
73	76	83	82	68	70		
		63	60				

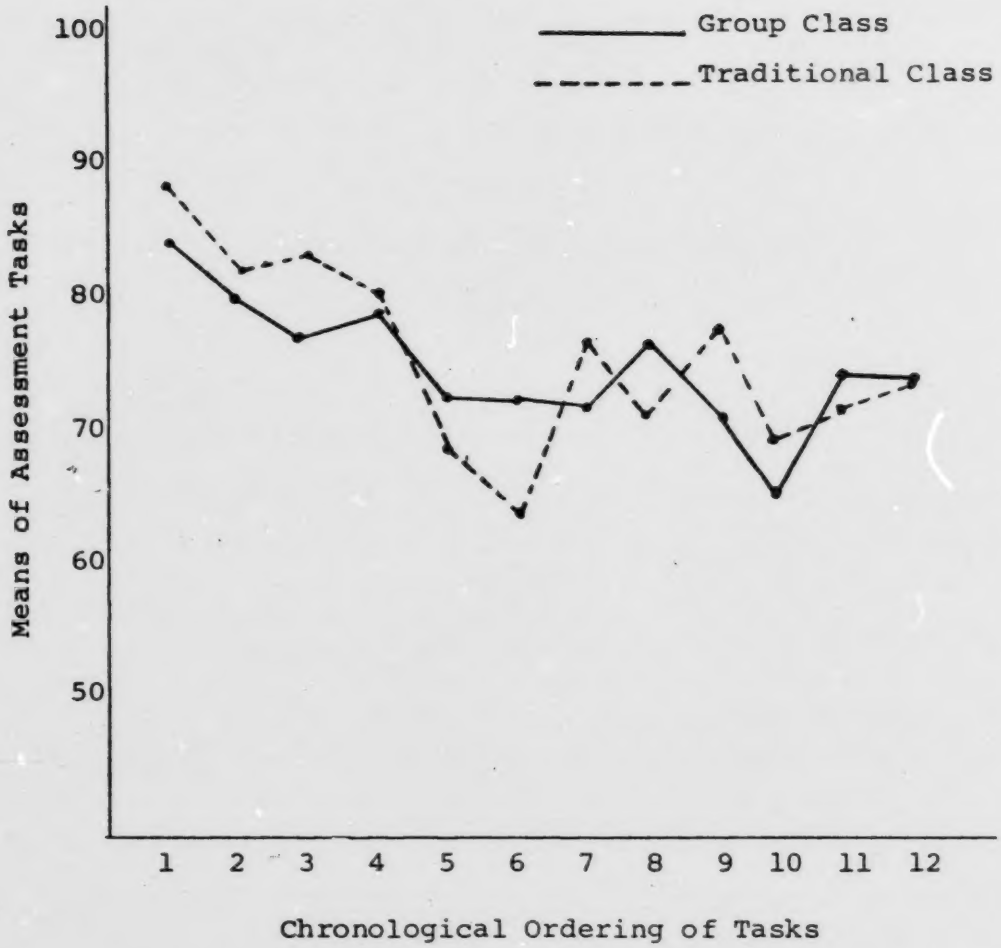

Figure 4.1

Means of Assessment Tasks by Condition and Chronology
--Good Counsel High School

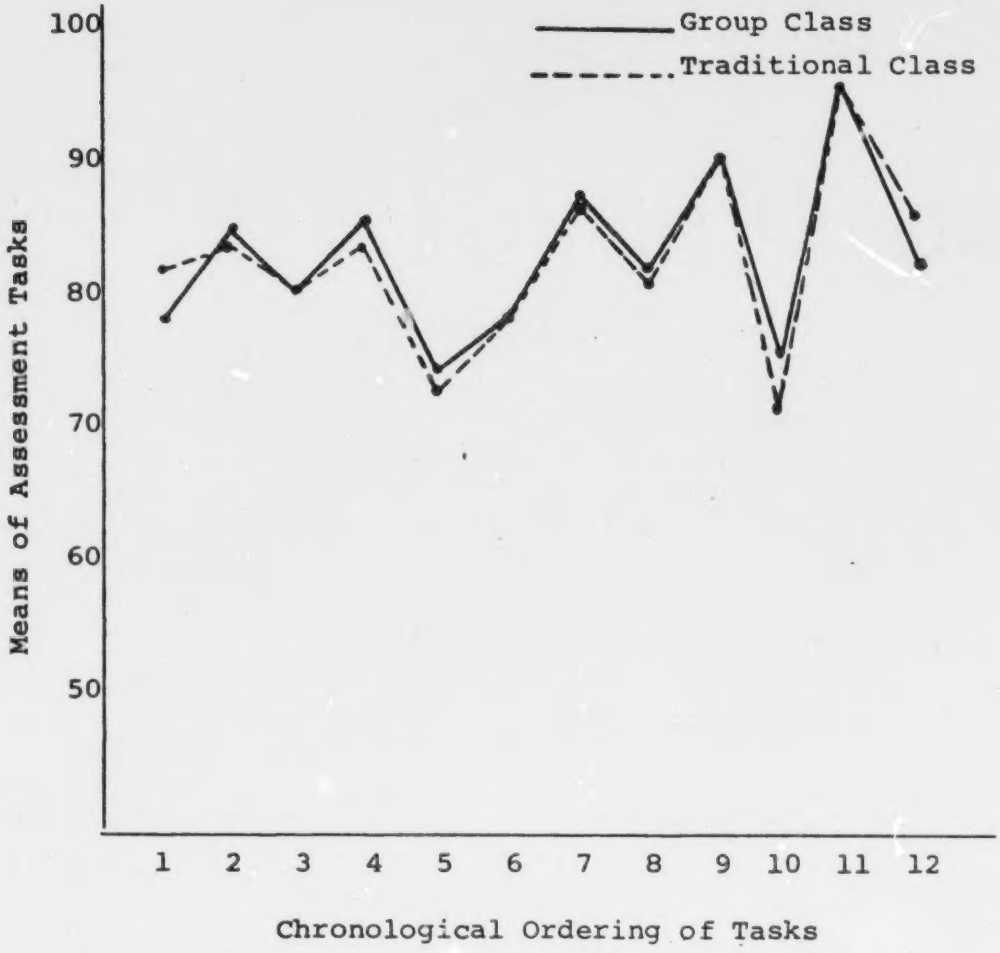

Figure 4.2

Means of Assessment Tasks by Condition and Chronology
--Bishop Ireton High School

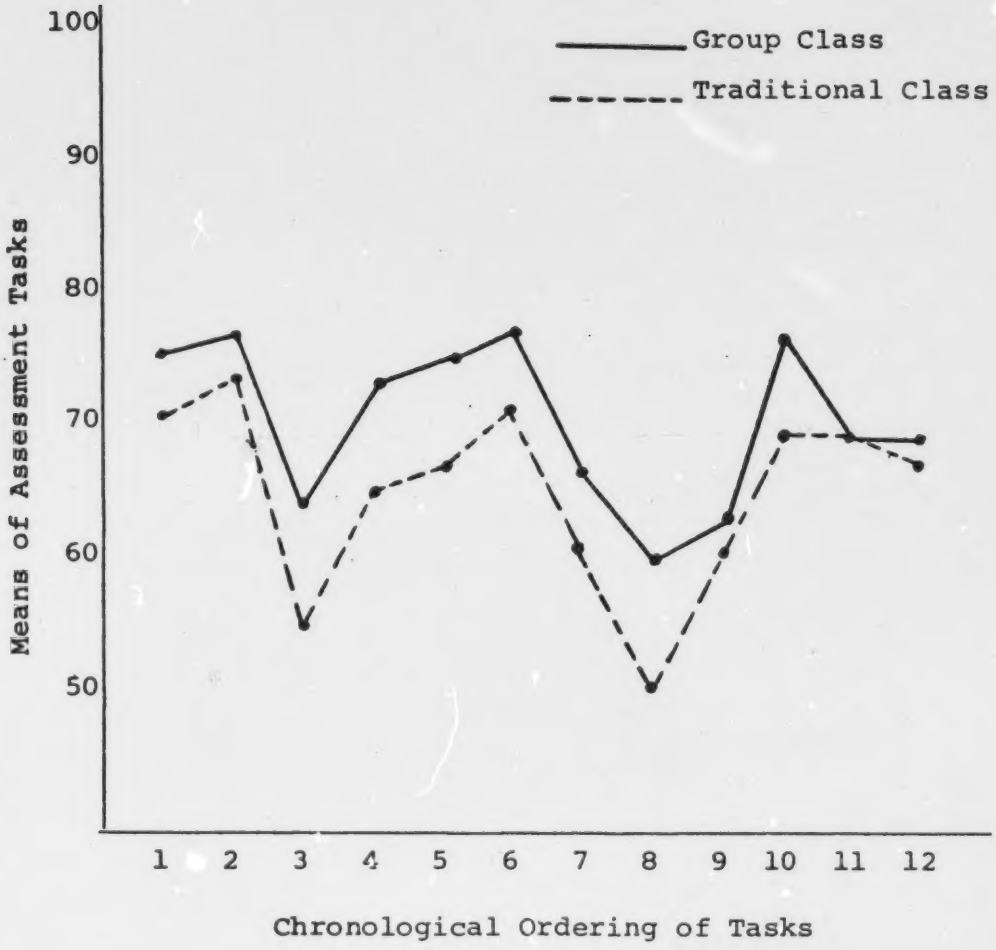

Figure 4.3

Means of Assessment Tasks by Condition and Chronology
--Salesianum High School

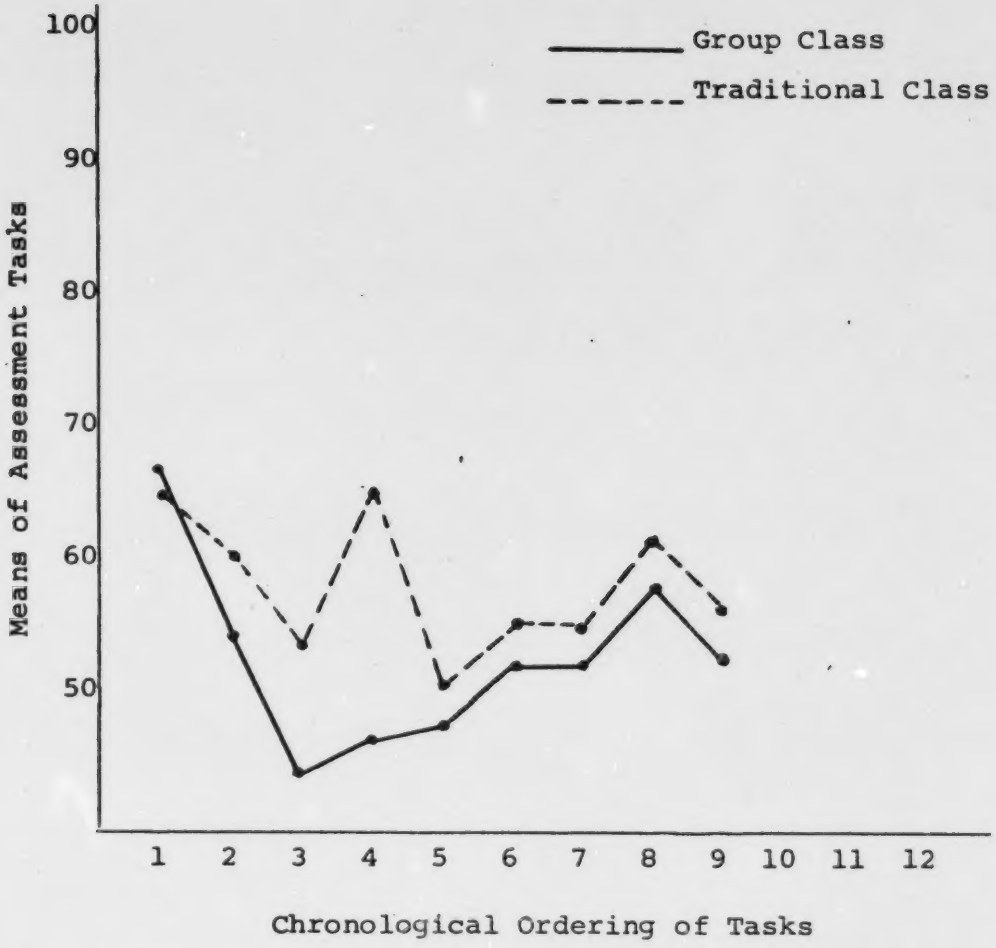

Figure 4.4

Means of Assessment Tasks by Condition and Chronology
--Northeast Catholic High School

The Statistic for the Kepner Test: t ratio

For the Kepner Mid-Year Algebra Achievement Test, the statistic used to test the null hypothesis that both samples came from the same population was the t ratio:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{s_{\bar{X}_1 - \bar{X}_2}} \quad ; \qquad (1.1)$$

where

$\bar{X}_1$ = mean of the group class,

$\bar{X}_2$ = mean of the traditional class,

μ_1 = mean of the population from which the group was selected,

μ_2 = mean of the population from which the traditional class was selected,

$s_{\bar{X}_1 - \bar{X}_2}$ = pooled estimate of the standard error of the difference between means.

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\left(\frac{\Sigma x_1^2 + \Sigma x_1^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \qquad (1.2)$$

where

$$\Sigma x_1^2 = \Sigma X_1^2 - \frac{(\Sigma X_1)^2}{n_1}$$

$$\Sigma x_2^2 = \Sigma X_2^2 - \frac{(\Sigma X_2)^2}{n_2}$$

X_1 = individual test score for group students,

X_2 = individual test score for traditional students,

n_1 = number of students in group class,

n_2 = number of students in traditional class.

When $n_1 = n_2 = n$, the formula reduces to

$$s_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{\Sigma x_1^2 + \Sigma x_2^2}{n(n-1)}} \qquad (1.3)$$

The alternative hypothesis was that there was a difference between the population means of the two classes, the group and the traditional.

Since this hypothesis is non-directional, a two-tailed test of significance was employed with the significance level set at .05.

Table 4.2 presents the raw scores and Figure 4.5 serves to illustrate the means achieved by each class in each school on the Kepner Test.

The analysis revealed that the alternate hypothesis had to be rejected for all four schools.

Table 4.3 presents a summary of the findings using the results of the Kepner Test. It includes the means, the standard deviations, the degrees of freedom, the t ratios obtained, the critical regions, and the decision reached in each case. The following statistical terms are used:

Table 4.2

Raw Scores of the Kepner Test for Both Classes in Each of the High Schools

Good Counsel		Bishop Ireton		Salesianum		Northeast Catholic	
Trad	Group	Trad	Group	Trad	Group	Trad	Group
9	26	17	8	21	16	8	6
9	24	8	11	26	11	15	13
11	17	34	25	21	22	13	13
29	13	15	23	9	20	9	12
16	11	24	23	25	14	12	7
17	16	18	14	34	22	12	10
13	22	10	13	27	18	9	7
14	16	24	22	21	28	12	7
14	16	19	22	29	28	10	12
7	17	17	19	13	31	9	9
10	16	20	28	20	22	9	8
28	20	18	21	20	19	13	12
21	8	14	21	17	38	10	8
25	17	19	17	20	26	13	9
19	11	25	25	11	19	11	3
18	11	12	17	20	32	8	9
21	24	16	22	27	22	11	7
8	11	18	11	23	19	11	12
14	12	30	28	14	28	8	4
10	16	19	16	17	37	9	9
21	23	18	19	21	24	11	14
9	18	27	24	30	31	9	7
17	22	9	13	13	15	9	11
23	17	23	10	33	28	9	13
17	19	19	29	12	24	11	14
17	19	19	29	12	24	11	14
16	23	25	15	19	11	11	8
15	16	12	19	26	17	8	10
23	19		9	31	37		
15	12		10	11			
21	18		10	12			
19	13		23				
22	12						
30	12						
				Mean			
17.00	17.09	18.89	18.29	20.76	23.54	10.37	9.41
				Standard Deviation			
6.19	4.93	6.23	6.18	7.00	7.53	1.86	2.96

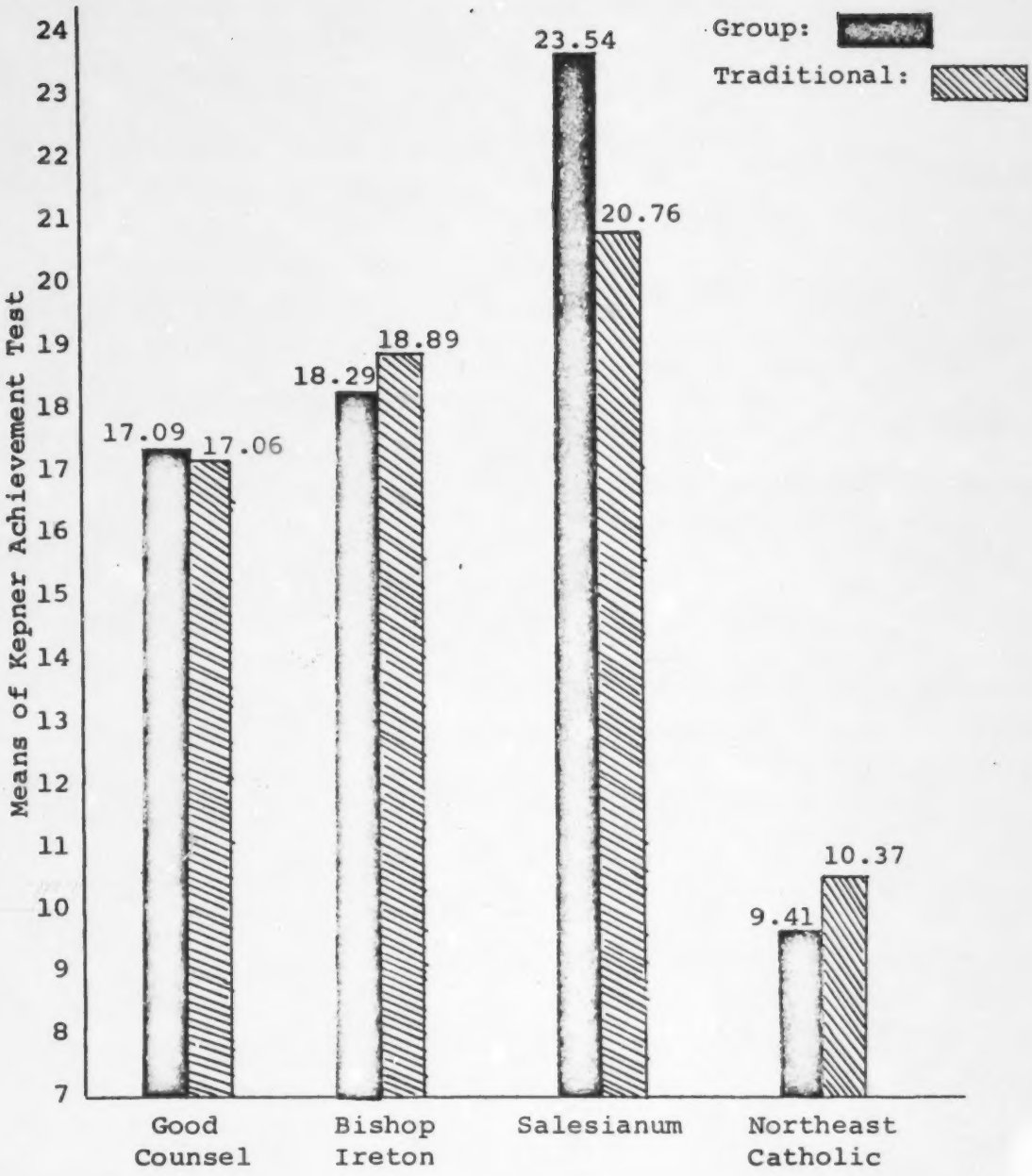

Figure 4.5

Means of Kepner Test by Schools and Condition

Table 4.3

Statistics Based on Kepner Achievement Test

School	Mean	Standard Deviation	Degrees of Freedom	t ratio	Critical Region	Decision
Good Counsel						
Traditional	17.06	6.19	64	0.0218	$\|t_{0.05}\| \geq 2.000$	Reject H_1
Group	17.09	4.93				
Bishop Ireton						
Traditional	18.89	6.23	56	-0.3681	$\|t_{0.05}\| \geq 2.021$	Reject H_1
Group	18.29	6.18				
Salesianum						
Traditional	20.76	7.00	56	1.45	$\|t_{0.05}\| \geq 2.021$	Reject H_1
Group	23.54	7.53				
Northeast Catholic						
Traditional	10.37	1.86	42	-1.5973	$\|t_{0.05}\| \geq 2.021$	Reject H_1
Group	9.41	2.96				

Null hypothesis (H_o): There is no significant difference between the population means of the traditional class and the group class on the Kepner Achievement Test.

Alternative hypothesis (H_1): There is a significant difference between the population means of the two classes on the Kepner Test.

Statistical Test: Student t-ratio two-sample case.

Significance level: $\alpha = 0.05$.

The Statistic for the Aiken-Dreger Scale: t ratio

Since a before-after design was employed in administering the Aiken-Dreger Mathematics Attitude Scale, the Student t ratio for correlated samples was appropriate:

$$t = \frac{\bar{D} - \mu_D}{s_{\bar{D}}}$$

where

$\bar{D} = \frac{\Sigma D}{n}$ = mean of the difference scores obtained by each student,

μ_D = mean difference of the population,

s_D = standard error of the mean difference,

$s_D = \sqrt{\frac{\Sigma d^2}{n(n-1)}}$,

$\Sigma d^2 = \Sigma D^2 - \frac{(\Sigma D)^2}{n}$,

n = number of students.

The null hypothesis was that there was no difference in the attitude of students toward mathematics before and after the eighteen weeks of instruction in the traditional and/or the group classes; i.e., $\mu_D = 0$.

The alternative hypothesis was that the attitude of students would be more favorable toward mathematics after the group class. This hypothesis was not posited for the traditional class.

Since this hypothesis is directional, i.e., $\mu_D < 0$, a one-tailed test of significance was employed, with the significance level set at .05.

Table 4.4 presents the raw scores obtained on the Aiken-Dreger Attitude Scale before and after the experiment. Figure 4.6 illustrates the means of these scores.

The analysis yielded the following results. In two schools, Good Counsel and Bishop Ireton, the null hypothesis was rejected for both classes, group and traditional, clearly signifying that significant attitude changes had occurred. However, in analyzing the scores of the classes at Salesianum High School, the alternative hypothesis was rejected for the traditional class: there had been no significant changes. For the group class, the difference scores were positive, a result of the fact that the scores in the before condition were higher than the scores in the

Table 4.4

Raw Scores of the Aiken-Dreger Attitude Scale for Both Classes Before and After the Experiment in Each of the High Schools

Good Counsel				Bishop Ireton				Salesianum				Northeast Catholic			
Trad		Group		Trad		Group		Trad		Group		Trad		Group	
B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A
68	76	74	67	42	57	37	41	39	60	40	26	45	45	61	68
63	49	54	64	53	56	79	69	50	72	51	58	38	61	67	69
55	63	30	47	57	78	78	76	55	58	72	32	46	42	47	34
50	38	46	31	38	72	61	99	65	58	76	51	44	43	66	63
33	66	36	20	53	65	61	61	77	82	76	66	41	61	83	73
53	63	28	58	34	51	36	25	48	62	75	65	71	66	59	74
76	90	62	73	68	69	75	76	57	57	69	51	71	79	63	60
70	91	32	52	61	79	61	61	52	35	65	59	44	31	57	55
70	73	54	26	54	42	66	94	61	59	48	52	66	72	52	69
56	42	75	68	67	74	48	27	74	60	77	80	51	48	59	56
54	46	58	71	50	79	51	56	61	63	68	69	93	84	67	79
84	79	72	69	53	57	74	76	66	64	47	58	83	78	62	64
56	75	69	77	44	74	69	74	57	35	56	51	79	81	76	71
43	63	54	57	29	76	44	53	66	78	71	72	63	61	82	51
66	76	57	91	54	63	42	60	44	20	62	50	55	65	42	57
67	74	58	66	74	86	79	86	54	54	68	48	55	68	40	47
76	60	76	64	65	58	58	57	50	53	80	81	61	77	43	37
50	63	39	55	52	71	76	74	40	28	86	99	50	44	50	55
60	47	64	58	35	68	37	50	54	61	87	87	62	70	53	55
55	48	67	64	50	65	62	66	59	61	55	64	49	58	57	34
60	92	67	92	55	56	39	62	55	50	78	78	61	80	43	64
36	89	59	75	62	64	45	97	45	85	73	68	59	53	62	65
81	85	66	63	71	79	45	51	41	52	46	55	80	75		
57	59	84	67	79	76	81	88	49	51	54	36	76	84		
41	42	68	60			48	58	51	59	63	56	53	77		
57	77	70	93			60	72	85	65	53	61				
72	72	73	97			40	77	65	61						
94	98	74	68			61	79	46	45						
56	81	37	52			67	76								
61	58	51	62												
		75	67												
		47	76												
		47	78												
		72	99												

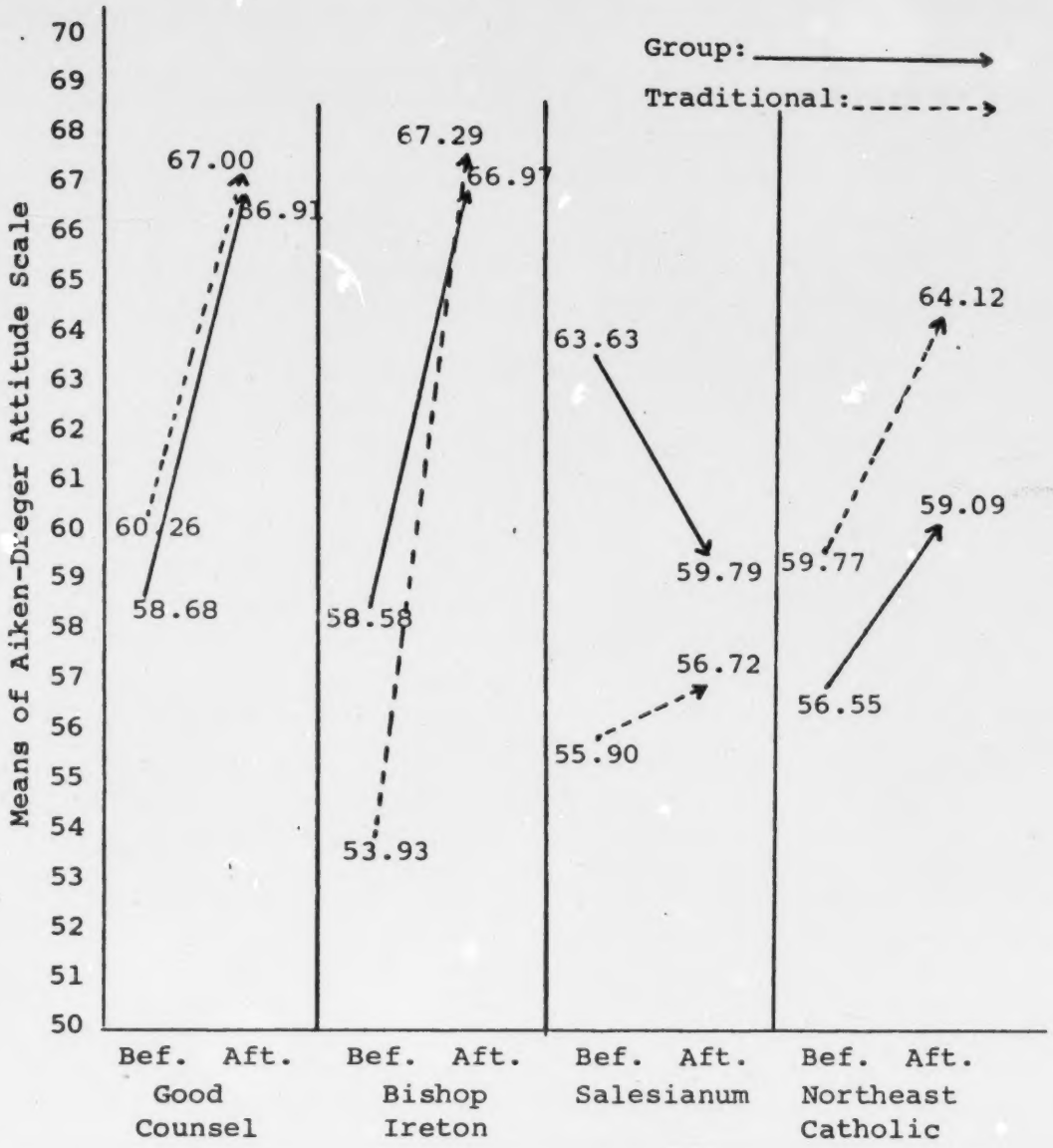

Figure 4.6

Means of Aiken-Dreger Scale by Schools and Condition
(Group-Traditional; Before-After)

after condition. Thus the alternative hypothesis was tested in the other direction and accepted at the .05 level, signifying a significant attitude change away from mathematics. Analysis of the Northeast Catholic scores demonstrated that the null hypothesis was rejected for the traditional class, signifying a significant attitude change, but the group class scores showed no such significant attitude change.

Table 4.5 presents a summary of the findings based on the Aiken-Dreger scale. It includes means, standard deviations, means of difference scores, degrees of freedom, t ratios, critical regions, and the decision reached in each case.

The following statistical terms were used:

Null hypothesis (H_o): There is no difference in the attitudes of students toward mathematics, before and after the experiment, in either the traditional or the group class.

Alternative hypothesis: The attitudes of the students in the group class will be more favorable toward mathematics after the experiment.

Statistical Test: Student t ratio for correlated samples.

Significance level: $\alpha = 0.05$.

Table 4.5

Statistics Based on Aiken-Dreger Attitude Scale

School	Mean		SD		Mean of Diff. Scores	df	$\underline{t}$ ratio	Critical Region	Decision
	Bef.	Aft.	Bef.	Aft.					
Good Counsel									
Traditional	60.26	67.00	13.76	16.87	- 7.17	29	-2.44	$t \leq -1.699$	Reject H_o
Group	58.68	66.91	15.04	16.43	- 6.85	33	-2.437	$t \leq -1.697$	Reject H_o
Bishop Ireton									
Traditional	53.93	67.29	12.63	10.74	-13.125	23	-4.526	$t \leq -1.714$	Reject H_o
Group	58.58	66.97	14.49	18.36	- 9.034	28	-3.155	$t \leq -1.701$	Reject H_o
Salesianum									
Traditional	55.90	56.72	11.02	14.36	- 0.857	27	-0.326	$t \leq -1.703$	Reject H_1
Group	63.63	59.93	14.93	16.68	+ 4.69	25	+1.892	$t \geq +1.708$	Reject H_o
Northeast Catholic									
Traditional	59.77	64.12	14.04	15.22	- 4.32	24	-2.06	$t \leq -1.711$	Reject H_o
Group	56.55	59.09	11.94	12.63	- 0.409	21	-0.153	$t \leq -1.721$	Reject H_1

Summary

The Student *t* ratio for a two-sample case was used to measure the results of the Kepner Achievement Test. Analysis indicated a rejection of the null hypothesis for all four high schools: there were no significantly higher achievement scores for the group classes in comparison with the traditional classes.

The Student *t* ratio for correlated samples was used to measure the results of the Aiken-Dreger Attitude Scale. In two schools, significant attitude changes were indicated, but for both the traditional and the group classes. In one school, a significant attitude change away from mathematics occurred in the group class and no significant attitude change occurred in the traditional class. In another school, significant attitude changes were indicated for the traditional class, but not for the group class.

CHAPTER V

CONCLUSIONS, INTERPRETATIONS, AND RECOMMENDATIONS

The conclusions deduced from the findings given in Chapter IV are reported in this chapter. In addition, interpretations and recommendations are given both as a result of the conclusions and as a result of individual teacher's comments concerning the experiment.

Conclusions

The purpose of the experiment described within this dissertation was to examine the effects of a teaching technique which would be an alternative to individual competition and individualized instruction. The technique involved lecture by the teacher for half the class time, with the other half devoted to within-group cooperation induced by between-group competition. It was undertaken in four high schools. The hypotheses were made that significantly higher achievement scores in mathematics would result from such a teaching method than from a traditional approach, and that

attitude towards mathematics would change significantly.

The hypothesis that significantly higher achievement scores would result was not supported in any one of the four schools where the experiment was conducted.

The hypothesis that a significant attitude change would occur was supported for the group classes in two of the high schools, but a significant change in attitude also occurred in the traditional classes in the same two high schools. In the other two high schools, this hypothesis was not supported. In one school, the traditional class evidenced a significant attitude change in favor of mathematics, but in the group class, no significant change was discovered. In the other school, the traditional class evidenced no significant attitude change, and the group class showed a significant attitude change, but in the opposite direction--away from mathematics.

Interpretation

Cooperative competition, the method employed to conduct the experiment, did not achieve the desired results. There were no significantly higher achievement scores in any of the high schools. The attitude changes seemed to be more dependent upon the teachers involved in two cases, since they occurred in both the traditional and the group classes.

In the other two cases, cooperative competition had no effect or an undesirable effect, an attitude change away from mathematics.

Discussion

Throughout the review of the literature, reported in Chapter II, a recurrent theme for cooperative group effectiveness was group interaction. Thompson[1] stressed this heavily in his assessment of the work previously done in cooperative and competitive group treatments. The cases he cited where significant results were recorded were learning tasks that could be accomplished only by means of cooperative interaction taking place.

If this is the key for significant achievement in group learning, the results of the experiment reported here should be examined in that light. How much interaction did in fact take place? What may have impeded the interaction that could have produced the desired results?

Variables Affecting Achievement

Half lecture--half group. The group class was structured in such a way that twenty-five minutes were given over to lecture by the teacher, and the other twenty-five minutes to group work. It may be asked whether such an

organization of class time permitted real group interaction. There was nothing intrinsic in the learning tasks undertaken by the groups that insured interaction. After the lecture by the teacher, the students were merely asked to practice skills and help one another in achieving some degree of competence. Perhaps too heavy a dependency on the teacher's lecture inhibited interaction to the degree that learning in the group class differed very slightly from the learning in the traditional class.

Dependency of cooperation on competition. On the face of it, what the investigator did in this experiment was to set up an environment where within-group cooperation was heavily dependent on between-group competition. For the subjects under consideration, the age level and low motivation seemed to justify using such an approach. It seems apparent that within-group cooperation generated by competition between groups did not insure sufficient interaction for significant learning to take place. There was cooperation in the sense of peer support and concern about a lagging performance by a group member, as well as pride in the accomplishment of the group as a whole, but interaction which would impel a group to probe a mathematical concept was non-existent.

Negative effects of competition. Between-group competition may have unwittingly induced some of the undesirable effects related to individual competition. Shaw[2] (discussed in Chapter II) found that competitive situations arouse stronger motivation to achieve than do cooperative situations, but this stronger motivation results in poorer performance. In the groups, peer pressure was sometimes very forceful for the improvement of the performance of a lagging group member. The desire to improve by such a group member was often very much in evidence. Yet, the improvement in many such cases was often minimal.

Low achievers. The subjects of the experiment were of low average to average ability. They were regarded as low achievers in mathematics by each of the four schools in comparison to their particular school populations. This designation was based on entrance examinations and past grades in mathematics in the elementary schools. Such students often look upon themselves as "losers" when it comes to mathematics. This investigator was of the opinion that such students might be particularly suited for the supportive effects of group work. It now seems obvious that such support may be beneficial in some respects, but support alone is not sufficient to improve learning. In a

group of low achievers, a situation can arise which is tantamount to "the blind leading the blind." In the group class, such groups became very dependent on the teacher, often looking at the classmates in their group with a sense of mistrust. When there is such a teacher dependency, very little group interaction can take place.

Group make-up. The subjects were placed into groups by the teacher on the basis of the Kepner test, the same test that was later used to measure achievement. High scorers on this test were mixed with low scorers so as to accomplish heterogeneous grouping by achievement. The Kepner test was not a good instrument to use for this purpose. It was an achievement test and presupposed a semester's work in Algebra. Thus many scores were the result of pure guesswork. However, even if an adequate test for arriving at heterogeneous ability grouping had been found, it may be asked whether a more suitable means of grouping should have been used. When groups are formed solely on the basis of heterogeneous ability, cooperation and interaction can become, at best, merely fortunate side-effects.

The discussion above has centered on possible explanations for the lack of significant achievement. Can similar reasoning be applied for the variable results with respect to attitude?

Variables Affecting Attitude

Teacher influence. It was pointed out in the preceding discussion that a twenty-five minute lecture by the teacher followed by twenty-five minutes of group activity may have resulted in only slight differences in learning in the group classes and the traditional classes. In two schools, the results of the attitude scale seemed to indicate that the teacher and not the group activity brought about significant attitude changes since the changes were evidenced in both classes. In another school, significant attitude changes were evidenced in the traditional class, but not in the group class. Here, too, the teacher's influence may have caused the changes and lack thereof, inasmuch as there was more teacher dominance in the traditional class than in the group class. In the last of the four schools, there were no significant changes in attitude in the traditional class and a change away from mathematics in the group class. Here, the teacher's influence may have been minimal, resulting in no change in attitude where he dominated. and

where he exerted even less influence, a change away from mathematics may have occurred because of other factors discussed below.

Group make-up. Since the groups were formed by the teacher on the basis of the Kepner test according to achievement, many variables were at work within the groups. For those students who for personal reasons were unsatisfied with the make-up of their group, attitude toward mathematics may have been similarly affected. This could apply also to those students who disliked working in groups, but preferred working alone. Finally, the sense of frustration of working in a group where "no one knows what's going on" may have had a deleterious effect on attitude toward mathematics.

Negative effects of competition. When competitive efforts between groups become very intense, the negative effects of individual competition can be realized. This may have occurred in some of the schools with the result that mathematics became the "fall-guy": the ultimate cause of peer pressure to improve, the reason for losing the respect of one's peers for not doing well, the game in which winning has been and always will be beyond the realm of attainment.

What then can be said about the method--cooperative competition? Are there any variations possible, and if so, what are the implications for future research? Can cooperative interaction be insured by other techniques? Should competition be retained or eliminated?

Cooperative Competition--An Evaluation

The experiment conducted by this investigator envisioned within-group cooperation which was heavily dependent on competition between groups. The method employed simply did not achieve the desired results. Thus, it is this investigator's opinion that the use of such a technique, without any variations, is at best questionable. Students in the group class did not achieve significantly higher scores than those in the traditional class, but they didn't achieve significantly worse ones either. However, attitude changes were so variable that the harmful side effects mentioned above must be considered.

Therefore, the following questions and resulting hypotheses are advanced for consideration.

Alterations for Possible Follow-up Studies

Discovery method. Davidson[3] used the small group-discovery method for learning calculus. His sample was

made up of volunteers who had A-B averages in mathematics in high school.

Is the discovery method suitable for low average to average students in mathematics selected randomly? Can a combination of the discovery technique and competition between groups overcome some of the shortcomings of the method employed in this experiment and produce significant achievement? Will such a combination guarantee group interaction to such an extent that working as a unit will eliminate the harmful effects of competition?

Hypothesis: A combination of discovery learning in small groups with competition between groups will bring about cooperative interaction and such interaction will eliminate the harmful effects of competition.

Student tutors. Eisenberg and Browne[4] used a method of student tutors to handle a large lecture in pre-calculus which was divided into small groups of 20 to 24 students. These small groups were further subdivided into groups of three or four. The tutors were undergraduates who were members of a secondary math-methods class. The authors reported significant improvement in skill development, improved attitude, and lower attrition rate when compared to a similar

class of the preceding year which had straight lecture.

Is it possible to use student tutors as an adjunct to the experiment described here? Can senior high school math students of better than average ability be enlisted for such a procedure? Would the assignment of a student tutor to each group enable him to serve as a coach-moderator if competition between groups were employed? Would such an assignment eliminate the frustration of a group in which "no one knows what's going on"? Would the tutor be able to downplay the negative effects of competition? Would significant achievement scores result?

Hypothesis: Small group cooperation stimulated by competition between groups will result in significant achievement and attitude change in a mathematics class in which student tutors of better than average ability are assigned to each group.

Learning guides. Is it possible to remove that teacher influence which fosters dependency by preparing learning guides for small groups of students? The learning guides would contain pretests and posttests, explain the matter, exhibit examples, point out reference books, require a series of problems to be solved, and have as an essential feature an appointment with the teacher. Would groups so

structured be able to generate sufficient interaction to promote learning? Would a class of thirty students so subdivided into small groups achieve significantly higher scores than another class not organized in this fashion? Would competition between groups add to or detract from the learning process?

Hypothesis: Small group learning through learning guides prepared by the teacher will bring about sufficient interaction so that significant learning takes place. Competition between groups will add another dimension to improvement in performance.

High achievers--highly motivated. With a group of high achievers, would the problems encountered in this experiment be eliminated? Would frustration at peers for "not knowing what's going on" be lessened? Are high achievers more competitive by nature, especially in an area in which they have often done well? In a group situation where groups compete with other groups, could this tendency if present be used to advantage? At the same time, would interaction among such a group be heavily dependent upon competition as it is for a low achieving group?

Hypothesis: Within-group cooperation stimulated by between-group competition is a more suitable learning device in

mathematics for high achievers than for low achievers. Such a method for high achievers will result in significantly better achievement.

Group make-up. What is the best way to compose a group? The method employed in this experiment was to assign students to groups on the basis of an achievement test. Would group make-up be better achieved through sociometric techniques? Would a trial period of two or three weeks be advisable after which students would be permitted to form their own groups? Would this bring about the undesirable effect of the better students in one group and the poorer students in another? Could the teacher then judiciously advise the groups of a better arrangement so as to insure a better mix? What would be the overall effect on performance if low achievers formed groups with which they were comfortable, and which were later rearranged by the teacher to insure heterogeneity?

Hypothesis: Small groups of low average to average achievers in mathematics formed by student preference and judiciously rearranged by the teacher will perform significantly better than small groups formed solely according to heterogeneous ability.

A final question should be asked in the light of the above hypotheses. Should between-group competition be eliminated altogether when small group learning is taking place? In Davidson's study of small group learning in calculus, competition between groups was discouraged. Yet, in answers to a questionnaire prepared by the teacher, seven out of the twelve students involved in the study reported that their groups competed with other groups, and only three students said that their group did not compete.[5] If it can be shown through a statistical study that competition between groups is a natural concomitance when groups work in the same classroom setting, then competition should be used to advantage, since it will be present anyway. If, on the other hand, competition between groups can be entirely eliminated, then its harmful effects would not be an issue.

Given the fact that many approaches to small group learning now exist in the schools, a research study could profitably demonstrate either the existence or non-existence of between-group competition when groups work in the same classroom. If the existence of competition is proven to exist, no matter what guidelines have been laid down, its effect on the functioning of groups should be further

examined. If the effects are harmful, ways to eliminate it should be explored.

The discussion thus far has concentrated on the negative aspects of the experiment and posed questions and offered hypotheses in that light. However, some positive results which were not measured statistically were observed. They are reported here as the comments of teachers and students who participated in the experiment: "The student's role in the classroom became more active than passive." "Most of the students in the group class knew immediately of at least one other student who was an 'authority' on the subject matter from whom he could get help outside class." "In the traditional class, the students cared only about their own grades. In the group class, students were concerned not only about their own grades, but also about their fellow students'." "One student who was by nature shy and an introvert opened up during the group work and became involved. Group interaction had a very positive influence on him." Finally, these comments were made in an exchange between two students and the teacher: "Do you realize that he had me on the phone for two hours last night explaining the homework?" "Well, that's what you're supposed to do--help your brother."

Summary

The present study has failed to demonstrate that cooperative competition has an effect on achievement in mathematics and has demonstrated that cooperative competition has variable effects on attitude toward the subject, depending upon the teacher's influence. For one-half the class time, an environment was set up for cooperative interaction, but cooperative competition did not guarantee its occurrence.

Various explanations for the non-occurrence or minimal occurrence of group interaction were offered. They were: only half the class time was devoted to group work; between-group competition of itself was insufficient to promote within-group cooperation; the harmful effects of individual competition were also side effects of group competition; the subjects were low achievers; the groups were made up by the teacher on the basis of an achievement test.

Variable attitude changes were attributed to the following possibilities: teacher influence, group make-up, and the negative effects of competition.

Several questions were asked and hypotheses posed as to the feasibility of adapting the method with several variations: discovery technique, use of student tutors,

use of learning guides, applying the method to high achievers, making up the groups in a different manner. The advisability of using competition as a motivator was also discussed. Finally, some positive aspects of the experiment were noted.

Footnotes--Chapter V

[1]G. Brian Thompson, "Effects of Cooperation and Competition on Pupil Learning," Educational Research, 15 (1972), 28-36.

[2]Marvin E. Shaw, "Some Motivational Factors in Cooperation and Competition," Journal of Personality, 26 (1958), 155-69.

[3]Neil A. Davidson, "The Small Group-Discovery Method of Mathematics Instruction as Applied in Calculus" (unpublished doctoral dissertation, University of Wisconsin, 1970).

[4]T. A. Eisenberg and J. B. Browne, "Using Student-Tutors in Precalculus Instruction," American Mathematical Monthly, 80 (1973), 685-88.

[5]Davidson, "The Small Group-Discovery Method," p. 297.

APPENDIX A

SYLLABUS: ALGEBRA I (ONE SEMESTER)

SYLLABUS FOR ALGEBRA I

(September, 1973--January, 1974)

1. Basic Concepts about numbers and numerals

   a. Positive numbers and zero
   b. Concepts of consecutive and successive
   c. "Greater than," "less than" concepts
   d. Coordinate of a point
   e. Betweenness
   f. Symbols for four fundamental operations
   g. Equations and statements
   h. Negative numbers plus all of the topics above
   i. Addition and its inverse
   j. Multiplication and its inverse
   k. Graphs and the number line
   l. Inequalities--concept--symbols

2. Variables, Expressions and Sentences

   a. Use of parentheses and brackets
   b. Translating verbal expressions to mathematical
   c. Grouping--order of operations
   d. Evaluating expressions
   e. Factors and coefficients
   f. Exponents--definition of, simple operations

3. Open sentences

   a. Equality and inequality (concept)
   b. Solving equations by inspection
   c. Using variables to write expressions
   d. Simple word problems
   e. Inequalities (solution by inspection)
   f. Graphing inequalities on the number line

4. Operations, Axioms, and Equations

   a. Basic axioms (reflexivity, symmetry, transitivity, closure)
   b. Basic Properties (commutative, associative, distributive laws)
   c. Properties of zero and one

5. Equations and Problem Solving

   a. Combining similar terms
   b. Addition and subtraction property of equality
   c. Multiplication and division property of equality
   d. Solving equations according to properties

6. Solving Inequalities through Properties

7. Addition and Subtraction of Polynomials

   a. Computation
   b. Solving equations using addition and subtraction of polynomials

8. Multiplication and Division of Polynomials

   a. Polynomials and exponents
   b. Multiplication of monomials
   c. Power of a product
   d. Multiplying a polynomial by a binomial
   e. Multiplying a polynomial by a polynomial
   f. Special products
      i. Binomial squares
      ii. Difference of squares
   g. Division: Monomial by monomial
      Polynomial by monomial
      Polynomial by binomial
   h. Factoring: simple explanation, simple examples

APPENDIX B

AN EXERCISE IN GROUP DYNAMICS:

DECISION BY CONSENSUS

DECISION BY CONSENSUS

Instructions: This is an exercise in group-decision making. Your group is to employ the method of Group Consensus in making its decision. This means that the prediction for each of the 15 survival items must be agreed upon by each group member before it becomes a part of the group decision. Consensus is difficult to reach. Therefore, not every ranking will meet with everyone's complete approval. Try, as a group, to make each ranking one with which all group members can at least partially agree. Here are some guides to use in reaching consensus:

1. Avoid arguing for your own individual judgments. Approach the task on the basis of logic.
2. Avoid changing your mind only in order to reach agreement and avoid conflict. Support only solutions with which you are able to agree somewhat, at least.
3. Avoid "conflict-reducing" techniques such as majority vote, averaging, or trading in reaching decisions.
4. View differences of opinion as helpful rather than as a hindrance in decision making.

On the "Group Summary Sheet" place the individual rankings made earlier by each group member. Take as much time as you need in reaching your group decision.

INSTRUCTIONS

Instructions: You are a member of a space crew originally scheduled to rendezvous with a mother ship on the lighted surface of the moon. Due to mechanical difficulties, however, your ship was forced to land at a spot some 200 miles from the rendezvous point. During re-entry and landing, much of the equipment aboard was damaged and, since survival depends on reaching the mother ship, the most critical items available must be chosen for the 200 mile trip. Below are listed the 15 items left intact and undamaged after landing. Your task is to rank order them in terms of their importance for your crew in allowing them to reach the rendezvous point. Place the number 1 by the most important item, the number 2 by the second most important, and so on through number 15, the least important.

Box of matches	_____	Stellar map (of moon's constellation	_____
Food concentrate	_____	Life raft	_____
50 feet of nylon rope	_____	Magnetic compass	_____
Parachute silk	_____	Signal flares	_____
Portable heating unit	_____	First aid kit containing infection needles	_____
Two .45 calibre pistols	_____	Solar-powered FM receiver-trans.	_____
One case dehydrated Pet Milk	_____		
Two 100 lb. tanks of oxygen	_____		
5 gallons of water	_____		

SCORING INSTRUCTIONS FOR DECISION BY CONSENSUS

The prediction is that the group product will be more accurate than the average for the individuals. The lower the score, the more accurate. A score of "0" is a perfect score.

Individual score: Each individual can score his own sheet. As you read aloud to the group the correct rank for each item, they simply take the difference between their rank and the correct rank on that item and write it down. Do this for each item and add up these differences--DISREGARD "+" and "-".

To get the average for all individuals, divide the sum of the individual scores by the number of individuals in the group. Compute the group score in the same way you computed each of the individual scores. If our hypothesis is correct, the group score will be lower than the average for all individuals.

POSSIBLE QUESTIONS FOR THE GROUP

1. Did the group really go by consensus? Or did we gloss over conflicts?
2. Did the group stay on the intellectual or task aspects or did we stop to examine our process to see how we could work more effectively?

3. How satisfied were we with the way the group worked? How efficient were we?

1 _____ 9

very poor excellent

4. How satisfied are you (as members) with tne group?
5. How much influence did you feel you had as an individual on the group decision?
6. Did the group listen to you? Ignore you?
7. Did you stay involved in the exercise or did you give up?
8. In what ways could you change or improve your interaction with others?

KEY

Little or no use on the moon	15 Box of matches
Supply daily food required	4 Food concentrate
Useful in tying injured together, help in climbing	6 50 feet nylon rope
Shelter against sun's rays	8 Parachute silk
Useful only if party landed on dark side	13 Portable heating unit
Self-propulsion devices could be made from them	11 Two .45 calibre pistols
Food, mixed with water for drinking	12 Dehydrated Pet Milk
Fills respiration requirement	1 Two tanks of oxygen
A principal means of finding directions	3 Stellar map
CO_2 bottles for self-propulsion across chasms	9 Life raft
Probably no magnetic poles; useless	14 Magnetic compass
Distress call when line of sight possible	10 Signal flares
Oral pills or injection medicine valuable	7 First aid kit with injection needles
Distress signal transmitter, possible commun. with mother ship	5 Solar-powered FM receiver-trans.
Replenished loss from sweating, etc.	2 Five gallons of water

GROUP SUMMARY SHEET

	1	2	3	4	Group Prediction
Box of matches					
Food concentrate					
50 ft. of nylon rope					
Parachute silk					
Portable heating unit					
Two .45 calibre pistols					
One case dehydrated milk					
Two tanks oxygen					
Stellar map					
Life raft					
Magnetic compass					
Signal flares					
First aid kit w/needles					
Solar-powered radio					
5 gallons of water					

BIBLIOGRAPHY

BIBLIOGRAPHY

Aiken, Lewis R., Jr. "Nonintellective Variables and Mathematics Achievement: Directions for Research." Journal of School Psychology, 8 (1970), 28-36.

_____. "Research on Attitudes Towards Mathematics." Arithmetic Teacher, 19 (March 1972), 229-34.

_____, and Dreger, R. M. "The Effect of Attitudes on Performance in Mathematics." Journal of Educational Psychology, 52 (1961), 19-24.

Baker, E. H. "A Pre-Civil War Simulation for Teaching American History." Simulation Games in Learning. Edited by S. S. Boocock and E. O. Schild. Beverly Hills, Calif.: Sage, 1968.

Bales, R. F. "Small Group Theory and Research." Sociology Today: Problems and Prospects. Edited by R. K. Merton, L. Broom, and L. S. Cottrell, Jr. New York: Basic Books, 1958.

Banghart, Frank W., and Spraker, Harold S. "Group Influence on Creativity in Mathematics." Journal of Experimental Education, 31 (1963), 257-63.

Barrish, Harriet; Saunders, Muriel; and Wolf, Montrose M. "Good Behavior Game: Effects of Individual Contingencies for Group Consequences on Disruptive Behavior in a Classroom." Journal of Applied Behavior Analysis, 2 (Summer 1969), 119-24.

Barton, W. A. "Group Activity Versus Individual Effort in Developing Ability to Solve Problems in First-year Algebra." Educational Administration and Supervision, 12 (1926), 512-18.

Bruner, Jerome. "The Continuity of Learning." Saturday Review of Education, February 1973, pp. 21-24.

Campbell, Lloyd. "Cooperation or Competition." Clearing House, 47 (May 1973), 563-64.

Cartwright, Dorwin, and Zander, Alvin, eds. Group Dynamics: Research and Theory. Evanston, Ill.: Row Peterson and Company, 1958.

Chapman, J. Crosby, and Feder, R. B. "The Effect of External Incentives on Improvement." Journal of Educational Psychology, 8 (1917), 469-74.

Clifford, M. M. "Effects of Competition as a Motivational Technique in the Classroom." American Educational Research Journal, 9 (Winter 1972), 123-37.

_____, and others. "Effects of Emphasizing Competition in Classroom Testing Procedures." Journal of Educational Research, 65 (January 1972), 234-38.

Coleman, James S. "Academic Achievement and the Structure of Competition." Harvard Educational Review, 29 (Fall 1959), 330-51.

Davidson, H. A.; Schriver, M. L.; and Peters, H. J. "Should Johnny Compete or Cooperate?" National Education Association Journal, 49 (October 1960), 30-32.

Davidson, Neil A. "The Small Group-Discovery Method of Mathematics Instruction as Applied in Calculus." Unpublished Ph.D. dissertation, University of Wisconsin, 1970.

Deutsch, Morton. "Cooperation and Trust: Some Theoretical Notes." Nebraska Symposium on Motivation, 1962. Edited by Marshall R. Jones. Lincoln, Nebr.: University of Nebraska Press, 1962.

_____. "An Experimental Study of the Effects of Cooperation and Competition upon Group Process." Human Relations, 2 (1949a), 199-232.

_____. "A Theory of Cooperation and Competition." Human Relations, 2 (1949), 129-51.

DeVries, David L., and Edwards, Keith J. "Learning Games and Student Teams: Their Effects on Classroom Process." American Educational Research Journal, 10 (Fall 1973), 307-18.

Dreikurs, Rudolph. Psychology in the Classroom. New York: Harper and Row, 1968.

Eisenberg, T. A. "The Integration of Modified Learner-Generated Sequences into the Development of a Behaviorally Stated Learning Hierarchy, as Applied in Mathematics Curricular Construction." Urpublished Ph.D. dissertation, University of Maryland, 1970.

_____, and Browne, J. B. "Using Student-Tutors in Precalculus Instruction." American Mathematical Monthly, 80 (1973), 685-88.

Gard, R. R. "Group Instruction with the Individual Touch." Clearing House, 46 (October 1971), 73-77.

Golembieski, Robert T. The Small Group. Chicago: University of Chicago Press, 1962.

Grossack, M. M. "Some Effects of Cooperation and Competition upon Small Group Behavior." Journal of Abnormal and Social Psychology, 49 (1954), 341-48.

Hare, A. Paul. "A Study of Interaction and Consensus in Different Sized Groups." American Sociology Review, 17 (1952), 261-67.

Hudgins, B. B. "Effects of Group Experience on Individual Problem Solving." Journal of Educational Psychology, 51 (1960), 37-42.

Hurlock, Elizabeth B. "The Use of Group Rivalry as an Incentive." Journal of Abnormal and Social Psychology, 22 (1927), 278-90.

Hyman, R. T. "Individualization: The Hidden Agenda." Elementary School Journal, 73 (May 1973), 412-23.

Jones, Thomas. "The Effect of Modified Programmed Lectures and Mathematical Games upon Achievement and Attitude of Ninth-Grade Low Achievers in Mathematics." Mathematics Teacher, 61 (1968), 603-607.

Julian, James W., and Perry, Franklyn A. "Cooperation Contrasted with Intra-Group and Inter-Group Competition." Sociometry, 30 (1967), 79-90.

_____; Bishop, Doyle W.; and Fiedler, Fred E. "Quasi-Therapeutic Effects of Intergroup Competition." Journal of Personality and Social Psychology, 3 (1966), 321-27.

Kirschenbaum, Howard; Napier, Rodney W.; and Simon, Sidney B. Wad-ja-Get? The Grading Game in American Education. New York: Hart Publishing Co., Inc., 1971.

Klugman, Samuel F. "Cooperative Versus Individual Efficiency in Problem-Solving." Journal of Educational Psychology, 35 (1944), 91-100.

Leuba, Clarence. "An Experimental Study of Rivalry in Young Children." Journal of Comparative Psychology, 16 (1933), 367-78.

Lindgren, Henry Clay. Educational Psychology in the Classroom. 3d ed. New York: John Wiley and Sons, Inc., 1967.

Lorge, I.; Fox, D.; Davitz, J.; and Brenner, M. "A Survey of Studies Contrasting the Quality of Group Performance and Individual Performance, 1920-1957." Psychological Bulletin, 55 (1958), 337-72.

MacKinnon, A. R. How Do Children Learn to Read? Toronto: Copp Clark, 1959.

May, Mark A., and Doob, Leonard W. Competition and Cooperation. A Report of the Sub-Committee on Competitive-Cooperative Habits. The Social Science Research Council, 1935.

McGrath, Joseph E., and Altman, Irwin. Small Group Research. New York: Holt, Rinehart and Winston, Inc., 1966.

McKeen, Ronald L. "A Model for Curriculum Construction through Observations of Students Solving Problems in Small Instructional Groups." Unpublished Ph.D. dissertation, University of Maryland, 1970.

Miller, L. Keith, and Hamblin, Robert L. "Interdependence, Differential Rewarding, and Productivity." American Sociological Review, 28 (1963), 768-78.

Mouly, George J. Psychology for Effective Teaching. 2d ed. Atlanta: Holt, Rinehart and Winston, Inc., 1968.

Myers, A. E. "Team Competition, Success and Adjustment of Group Members." Journal of Abnormal and Social Psychology, 65 (November 1962), 325-32.

Philips, B. N., and DeVault, M. V. "Evaluation of Research on Cooperation and Competition." Psychological Reports, 3 (June 1957), 289-92.

Rabbie, Jacob M., and Wilkens, Gerard. "Intergroup Competition and Its Effect on Intragroup and Intergroup Relations." European Journal of Social Psychology, 1 (1971), 215-34.

The School Boys of Barbiana. Letter to a Teacher. Translated by Nora Rossi and Tom Cole. New York: Vintage Books, Random House, Inc., 1971.

Shaw, Marjorie E. "A Comparison of Individuals and Small Groups in the Rational Solution of Complex Problems." American Journal of Psychology, 44 (1932), 491-504.

Shaw, Marvin E. Group Dynamics: The Psychology of Small Group Behavior. New York: McGraw-Hill Book Company, 1971.

Sims, Verner Martin. "The Relative Influence of Two Types of Motivation on Improvement." Journal of Educational Psychology, 19 (1928), 48-84.

Slater, P. E. "Contrasting Correlates of Group Size." Sociometry, 21 (1958), 129-39.

Taylor, Donald, and Faust, William. "Twenty Questions: Efficiency in Problem Solving as a Function of Size of Group." Journal of Experimental Psychology, 44 (1952), 360-68.

Thelen, H. A. "Group Dynamics in Instruction: Principle of Least Group Size." School Review, 57 (1949), 139-48.

Thompson, G. Brian. "Effects of Cooperation and Competition on Pupil Learning." Educational Research, 15 (1972), 28-36.

Turner, V. D.; Alders, C. D.; Hatfield, F.; Croy, H.; and Sigrist, C. "A Study of Ways of Handling Large Classes in Freshman Mathematics." American Mathematical Monthly, 73 (1966), 768-70.

Vincent, M. M. "Another Look at Competition in the Classroom." Educational Forum, 26 (March 1962), 289-92.

Whittemore, Irving C. "The Influence of Competition on Performance: An Experimental Study." Journal of Abnormal and Social Psychology, 19 (1925), 236-54.